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Analyzing Mathematics Teachers' Questions on the Concepts of Mean, Median, and Mode using TinkerPlots: A Focus on the Type of Mathematical Knowledge

Kenan GÖKDAĞ [a] *, Meriç ÖZGELDİ [b], Nazım ÖZKAN [c]

[a] *Akdeniz Middle School, Mersin, 33770, Turkey*

[b] *Mersin University, Mersin, 33770, Turkey*

[c] *Mehmet Akif Ersoy Secondary School, Balikesir, 10010, Turkey*

Abstract

The purpose of the study was to investigate how the different types of mathematical knowledge, as reflected in mathematics teachers' questions regarding the concepts of mean, median, and mode, evolve through the use of TinkerPlots. Six mathematics teachers participated in a 16-hour TinkerPlots training. The mathematics teachers' questions were analyzed in terms of contextual, conceptual, and procedural knowledge both before and after the training. The findings revealed that, before the training, the emphasis was largely on learning and applying the procedural aspects of the concepts of mean, median, and mode. Following the training, however, there was a notable shift towards questions that not only encompassed procedural knowledge but also fostered conceptual and contextual understanding. This study underscored the importance of TinkerPlots training in shaping teachers' approaches to statistical thinking, thereby enhancing their professional development. Educational implications for teachers' professional development were discussed.

Keywords: Mathematics teachers; mathematical knowledge; statistical thinking; statistical concepts; TinkerPlots

1. Introduction

1.1. Introduction to the problem

Statistics, both as a scientific discipline and a profession, enables students to understand and analyze the world in a more systematic way. It is regarded as a

* Corresponding author name. ORCID ID.: https://orcid.org/0000-0000-0000-0000
E-mail address: author@institution.xxx

comprehensive intellectual and academic method that encompasses fundamental concepts such as data, variability, and probability (Moore, 1997). Comparative analyses of curricula from different countries emphasize the importance of students understanding basic statistical concepts like mean, median, and mode before entering secondary education (Australian Education Council, 1994; National Council of Teachers of Mathematics [NCTM], 2000; School Curriculum and Assessment Authority & Curriculum and Assessment Authority for Wales, 1996). At this foundational stage, the depth of teachers' content knowledge plays a critical role in students' understanding of these concepts (Groth & Bergner, 2006). The tools and strategies used in teaching these statistical concepts play an equally important role in supporting the learning process for both teachers and students. Specifically, statistical measures such as mean, median, and mode are among the essential knowledge components that elementary mathematics teachers are expected to possess (Groth & Bergner, 2006).

Mewborn (2003) stated that training both teachers and students with effective methods and techniques can introduce new perspectives on thinking. Over time, various strategies and tools have been developed to enhance teaching and learning opportunities for educators and students alike. In this context, dynamic statistical software tools have emerged as valuable resources for fostering a deeper understanding of data interpretation, variability, and probability. These tools not only enhance students' comprehension of statistical methods but also enrich educators' teaching practices (Lajoie, 1993; Heid, 1995). Software such as Fathom, TinkerPlots, Probability Explorer, and Chance-Maker Microworld plays a significant role in transforming students' attitudes toward learning statistics. Specifically, TinkerPlots stands out as an effective tool for enabling students to explore and understand statistical concepts (Konold & Miller, 2001). However, taking full advantage of the benefits offered by such tools depends on teachers' ability to develop effective questioning techniques.

A significant challenge in teaching statistical concepts lies in the types of questions teachers design to facilitate students' reasoning and comprehension. Research indicates that teachers tend to emphasize procedural knowledge in their questioning techniques, which restricts opportunities to deeply examine the conceptual and contextual aspects of statistical concepts (Biggs & Tang, 2011). Wood (2002) highlights that differences in students' thinking and reasoning processes are closely related to the types of questions posed by teachers. Questions that integrate contextual, conceptual, and procedural dimensions can provide students with a more holistic perspective on statistical concepts by enabling them to make meaningful connections with real-world data (Moyer & Milewicz, 2002).

To overcome these challenges, professional development programs play a critical role in equipping teachers with the skills to create more diverse and meaningful questions. Research indicates that integrating dynamic tools such as TinkerPlots into teacher education not only enhances content knowledge but also enriches teachers' pedagogical strategies (Fitzallen, 2013; Koparan & Kaleli-Yılmaz, 2014; Watson, 2008). Such programs enable teachers to go beyond traditional teaching approaches and adopt methods that foster higher-order thinking, data analysis, and interpretation (Anderson & Krathwohl, 2001). Through TinkerPlots, teachers gain access to tools for exploring and visualizing the dynamic nature of data, allowing them to formulate more effective questioning techniques that enhance students' understanding of statistical concepts.

Despite the importance of statistical literacy, many teachers predominantly focus their questions on procedural tasks, limiting opportunities to explore conceptual or contextual foundations. This narrow focus makes it difficult for students to appreciate the real-world applicability and dynamic nature of statistical concepts. Furthermore, professional development opportunities aimed at improving teachers' ability to craft comprehensive and multidimensional questions remain limited. In this context, examining how dynamic tools like TinkerPlots transform teachers' questioning practices and enhance their ability to integrate procedural, conceptual, and contextual knowledge into teaching becomes essential. Building on this, the purpose of the study was to investigate how the different types of mathematical knowledge reflected in mathematics teachers' questions regarding the concepts of mean, median, and mode evolve through the use of TinkerPlots. This study focuses on the following research question:

How does the mathematical knowledge demonstrated in the questions formulated by mathematics teachers about the concepts of mean, median, and mode evolve following TinkerPlots training?

1.2. Theoretical Framework

Knowledge is a fundamental component in mathematics education, directly influencing both learning outcomes and teaching strategies (Shulman, 1986). Researchers have conceptualized knowledge in various ways, associating it with different types (e.g., declarative, procedural) and attributes, such as what knowledge and why knowledge (Alexander et al., 1991; de Jong & Ferguson-Hessler, 1996). These types of knowledge enable students not only to recall information but also to apply it effectively in meaningful contexts. De Jong and Ferguson-Hessler (1996) emphasize that knowledge is not merely a collection of abstract fragments but comprises structured frameworks that support the execution of specific tasks. Thus, knowledge is understood within a cognitive framework as both discrete elements and an organized structure that underpins learning and application.

Within this framework, knowledge is highlighted as a means of enabling individuals to engage deeply with mathematical concepts, beyond merely recalling isolated facts, thereby enhancing their problem-solving abilities (Bransford et al., 2001). The ability of students to form connections between concepts and utilize these connections to address novel problems is critical for long-term learning retention. In mathematics education, knowledge is regarded as an organized structure encompassing various forms (e.g., conceptual, procedural, and contextual), which guides students through mathematical reasoning processes (Aydın & Özgeldi, 2019; Hiebert & Lefevre, 1986).

The importance of understanding how different types of knowledge—contextual, conceptual, and procedural—work together in mathematics is emphasized (Aydın & Özgeldi, 2019). These types of knowledge not only equip students and teachers with the skills to accurately perform tasks but also enable them to connect mathematical concepts with real-world applications. Aydın and Özgeldi (2019) argue that understanding the relationships between different types of knowledge moves students beyond merely learning procedures, offering more meaningful learning experiences. Furthermore, the integration of these knowledge types not only supports the execution of mathematical tasks but also fosters the meaningful application of knowledge (Rittle-Johnson & Koedinger, 2005).

1.2.1. Types of Mathematical Knowledge

Researchers have categorized mathematical knowledge into various types, with conceptual, contextual, and procedural knowledge identified as key contributors to mathematical understanding and task performance (de Jong & Ferguson-Hessler, 1996; Hiebert & Lefevre, 1986; Rittle-Johnson & Koedinger, 2005). These types of knowledge are integral to fostering

statistical thinking, which involves the ability to reason about data, understand variability, and draw meaningful conclusions. They enable students to develop strategies, interpret data, and select the appropriate type of knowledge based on the requirements of a specific task (Alexander et al., 1991).

Conceptual knowledge supports statistical thinking by helping learners understand the principles and relationships underlying statistical measures, such as how variability affects the mean or why the median may be a better measure of central tendency in certain contexts. Contextual knowledge enhances this by allowing students to connect these concepts to real-world scenarios, thereby developing a more nuanced understanding of data's significance. Procedural knowledge, in turn, ensures the accurate computation of statistical measures, providing a foundation upon which deeper reasoning and interpretation can occur. Aydın and Özgeldi (2019) explore the challenges prospective teachers face in integrating these types of knowledge, highlighting the importance of a balanced approach. Understanding how conceptual, contextual, and procedural knowledge interact not only enhances mathematical comprehension but also equips teachers to cultivate statistical thinking in their students—an essential skill for navigating data-driven decision-making in professional and everyday contexts. The following sections analyze the role of each knowledge type and its importance in understanding statistical concepts such as mode, median, and arithmetic mean.

1.2.1.1. Contextual Knowledge

Contextual knowledge refers to the ability to understand how mathematical concepts apply to real-world situations. For statistical measures such as mode, median, and arithmetic mean, contextual knowledge is crucial for understanding their use in societal, economic, or scientific contexts (Gravemeijer & Doorman, 1999). This type of knowledge allows students and teachers to interpret these concepts within meaningful scenarios rather than as abstract calculations (Saxe, 1988).

Contextual knowledge involves the skill of identifying the most representative statistical measure for a data set, depending on the situation's requirements (Garfield & Ben-Zvi, 2008). For instance, recognizing that the median provides a more appropriate representation than the mean in cases involving extreme outliers highlights the importance of contextual knowledge. This understanding is vital for interpreting real-world issues, such as income distribution, where the median often offers a more balanced representation of central tendency compared to the mean (Groth & Bergner, 2006).

Teachers with contextual knowledge can help students recognize that certain measures may be more meaningful depending on the nature and content of a dataset. This understanding fosters students' statistical literacy and promotes critical thinking. Tools like TinkerPlots support teachers in engaging with real-world data analyses, enhancing their contextual knowledge (Konold & Miller, 2005). This enriched knowledge can then be utilized to create meaningful and interactive learning environments for students (Freudenthal, 1991; Sáenz, 2009).

1.2.1.2. Conceptual Knowledge

Conceptual knowledge involves understanding the principles, relationships, and structures underlying mathematical concepts (Hiebert & Carpenter, 1992). In the context of statistical measures such as mode, median, and arithmetic mean, conceptual knowledge enables students and teachers to view these measures not merely as definitions but as tools for interpreting data distributions. This type of knowledge is critical for understanding how these measures interact with data sets. For example, recognizing that the median is less affected by outliers compared to the mean allows students to appreciate why the median may be a more appropriate measure of central tendency in certain situations (Groth & Bergner, 2006).

Moreover, conceptual knowledge helps students connect statistical measures with practical applications (Wild & Pfannkuch, 1999). Understanding the median as a measure that divides a data set into two equal parts, for instance, supports discussions on distribution and equity, such as in analyzing income inequality. This perspective encourages students to think of statistical concepts not only as calculations but also as tools relevant to everyday life. Tools like TinkerPlots further aid teachers in deeply exploring conceptual knowledge, enhancing their ability to deliver this knowledge effectively and flexibly in the classroom (Rittle-Johnson & Alibali, 1999).

Conceptual knowledge also helps students grasp the reasoning behind mathematical procedures, enabling them to see mathematics as a meaningful process rather than a series of mechanical steps. By focusing on the mathematical significance behind operations rather than abstract procedures, students develop stronger problem-solving and critical-thinking skills, which are essential for deeper mathematical understanding (Hiebert & Lefevre, 1986).

1.2.1.3. Procedural Knowledge

Procedural knowledge encompasses the steps and rules required for accurately calculating mathematical measures (Hiebert & Lefevre, 1986). For concepts such as mode, median, and arithmetic mean, procedural knowledge involves understanding the specific calculation processes: recognizing the mode as the most frequently occurring value, the median as the middle value in an ordered data set, and the arithmetic mean as the sum of values divided by their count. Mastery of these steps ensures that learners can apply these measures correctly (Star, 2005).

While procedural knowledge is essential for accurate computations, it becomes more effective when integrated with conceptual knowledge (Rittle-Johnson & Alibali, 1999). For instance, calculating the mean accurately requires not only following the steps but also understanding the significance of the mean within the context of the data set. Teachers play a critical role in facilitating this integration by explaining the reasoning behind each procedure, demonstrating how procedural steps fit into a broader mathematical framework (Hiebert & Lefevre, 1986).

Interactive tools such as TinkerPlots provide teachers with opportunities to practice statistical procedures in dynamic environments, reinforcing accuracy and confidence (Konold & Miller, 2005). Such training allows teachers to model procedural steps effectively for students while helping learners develop a better understanding of when and how to apply each measure. Procedural knowledge thus enables students to perform data analysis accurately and thoughtfully, fostering both precision and awareness.

To sum up, contextual, conceptual, and procedural knowledge play an integrative role in teaching and learning statistical concepts such as mode, median, and arithmetic mean. Contextual knowledge helps learners connect mathematical principles to real-world applications; conceptual knowledge deepens their understanding of the underlying principles and relationships; and procedural knowledge ensures accurate computation. Programs like TinkerPlots, which focus on these knowledge types, might empower teachers to provide students with a comprehensive understanding of statistical concepts, facilitating their meaningful application in various contexts.

2. Method

This section provides a detailed explanation of the research design, participants, and data collection and analysis. Each stage was meticulously planned and implemented to enhance the reliability and validity of the research.

2.1. Research Design

The data were gathered during a 16-hour TinkerPlots training program conducted in collaboration with the university and the national education directorate. In the training, mathematics teachers were introduced to the basic functions of the TinkerPlots software and engaged in hands-on activities related to its use in statistical analysis. The training was designed as a learning environment with the goal of gathering high-quality and comprehensive data. In this context, a case study method, known for its ability to facilitate a systematic, detailed, and in-depth examination of a specific context or phenomenon (Patton, 2018), was employed. This design allowed for a comprehensive analysis of the mathematics teachers' experiences during the training process.

2.2. Participants

Criterion sampling involves including individuals who meet specific predefined criteria, ensuring that the collected data are more directly aligned with the research questions (Patton, 2018). The participants were selected using criterion sampling, a purposeful sampling method in this study. 12 teachers who were eager to improve their statistical analysis skills participated in the training. Two primary criteria were used in the selection of participants: (1) they had successfully completed courses in statistics and probability during their graduate studies (participants were enrolled in a master's degree program) and (2) they had at least two years of teaching experience in mathematics education. These criteria ensured that participants had a foundational knowledge of statistics and familiarity with software usage, facilitating a more effective examination of the learning and application areas emphasized during the study. A total of six teachers who met these criteria and voluntarily agreed to participate were included in the study.

2.3. Data Collection and Analysis

The data were derived from worksheets administered by the teachers both prior to and following the training. These worksheets contained questions addressing the statistical concepts of mode, median, and arithmetic mean, serving as the primary source of data to evaluate the changes in the teachers' questions. Figure 1 illustrated the questions asked by the participants before and after the TinkerPlots.

Teacher X

Worksheet created before the training

WORKSHEET 1

NAME	WEIGHT	HEIGHT
Aybüke	40	150cm
Berkay	52	152cm
Kaan	53	151cm
Berk	45	147cm
Ecem	40	145cm
Çağan	45	147cm
Doruk	52	151cm
Aleyna	45	147cm
Elif	42	146cm

The table on the right shows the weight and height of the students in a class. Answer the following questions according to these data.

1- Find the weight average of the class?

2- Calculate the mode of the lengths?

3- Find the median of the weights?

4- Does the median change when Elif is removed from the group?

5- Find the mean of the height of male and female students separately?

Worksheet created after the training

NAME	WEIGHT	HEIGHT (CM)	GRADE POINT AVERAGE	FAMILY MONTHLY INCOME
AHMET	65	170	30	₺1000
VELİ	60	150	47	₺1500
HASAN	70	165	95	₺5000
AYŞE	49	152	75	₺2500
FATMA	53	159	70	₺2200
SERGEN	62	180	90	₺6500
HAYRİYE	65	177	85	₺3500
BURAK	59	160	57	₺1800
NESLİHAN	45	148	65	₺2000
FATİH	90	170	90	₺4500

In the table on the right, data are collected on the weight, height, grade point average and monthly family income of the students in a class. According to these data, the following questions are asked.

1) Find and compare the weight averages of male and female students in the class separately?

2)Compare the grade point averages of male and female students. Which gender group is more successful?

3) Compare the grade point averages of students with a monthly income above and below 3000₺. Discuss whether financial income affects grade point averages.

4) When Sergen leaves this class, analyse the change in mean, mode and median of the data?

5)Which unit of measurement should we use to comment on the data?

Teacher Y

Worksheet created before the training

WORKSHEET 1

The amounts of fruit juice, toast, water and chocolate sold in the school canteen in 1 week are given.

	FRUIT JUICE	TOAST	WATER	CHOCOLATE
GAMZE	8	4	5	4
MERVE	9	3	5	2
GÜLCİHAN	5	1	6	7
NAZIM	6	0	2	3

1) Find the average for the most consumed product. Explain how you found it.
2) Find the average for the least consumed product. Explain how you found it.
3) Find the median for the most sold product.
4) Find the mode for the data. Explain how you found it.

Worksheet created after the training

WORKSHEET 2

1)

	0-5 AGE	5-10 AGE	10-15 AGE
A	10	17	6
B	27	5	25

The number of students aged 0-5, 5-10, 10-15 in classes A and B are given.

a) How will the average age change if 2 students aged 5-10 in class B move to class A?

b) How will the average age change if 3 students aged 10-15 in class A leave the class?

2) The grades of the students in a class in the maths exam are given in the table.

30, 40, 50, 55, 25, 27, 40, 38, 100, 50, 40, 40, 10, 15, 40, 50

a) Find the median. Explain how it was found.

b) Based on the grades, what is the mode of the data set? How does the mode change if 4 students with a grade of 30 participate?

c) How does the median change if 5 students with grades 30, 40, 55, 30, 20 from either class participate?

Figure 1. Sample questions in worksheets before and after the TinkerPlots training with mode, median and arithmetic mean

The data analysis process was systematically carried out in four main stages. The first stage, the coding process, began with a content-based examination of each item in the worksheets, specifically the participants' responses. In this phase, items were independently coded by two experts and two mathematics teachers from the research team. During coding, each item was classified within the framework of contextual, conceptual, and procedural knowledge types, as defined by Rittle-Johnson and Koedinger (2005). A descriptive analysis approach was utilized to analyze the data. This method entails interpreting and organizing the collected data according to predefined themes, ensuring that participants' responses are accurately reflected throughout the analysis process (Creswell, 2013). Since each item could be associated with more than one type of knowledge depending on its content, multiple categorizations of the items were allowed. This method enabled a more comprehensive and in-depth evaluation of the relationships between knowledge types. The independent coding process formed the first step of the analysis by maintaining the diversity and accuracy of individual evaluations.

After the coding process, the second stage, coding reliability and consensus, was initiated. Following the independent coding by the coders, the categories assigned to each item were compared. In cases of discrepancies among the coding results, these differences were thoroughly discussed during regularly held meetings. The rationale behind assigning a particular category to each item was explicitly expressed and debated. By the end of this process, full consensus was reached on all items. The third stage, categorization and classification, served as the foundation for organizing and analyzing the data obtained during the coding process. In this stage, the items in the worksheets were categorized based on contextual, conceptual, and procedural knowledge types. In the contextual knowledge category, the appropriateness of the questions to real-life contexts and how they were utilized in such contexts were evaluated. The conceptual knowledge category examined whether statistical concepts were correctly and meaningfully expressed, while the procedural knowledge category analyzed whether mathematical procedures were applied correctly. Each item was associated with one or more types of knowledge depending on its content. This classification revealed the distribution of knowledge types in the questions prepared by teachers.

The final stage, presenting findings specific to categories, involved organizing and reporting the data obtained during the analysis process. The findings from the coding and classification stages were detailed according to the identified knowledge types. Sample questions for each category and the analysis results of these questions were presented in connection with the themes. Examples of categorized items were included in Table 1. contributed to the visualization of the findings, enhancing the comprehensibility of the analysis process.

Table 1. Coding examples of teachers' question samples

Statistical concepts	**Teachers' questions**	**Type of knowledge**
Mean	What is the average age of the students?	Procedural
Mean	Are these measures of central tendency consistent? Why?	Conceptual
Mean	Find the average for the least consumed product. Explain how you found it.	Contextual & Conceptual
Median	What is the median?	Procedural
Median	Find the median. Explain how it was found.	Procedural & Conceptual
Median	Calculate the median of the lengths.	Contextual
Mode	What is the mode?	Procedural
Mode	Calculate the mode of the lengths.	Procedural & Contextual
Mode	Find the mode for the data. Explain how you found it.	Conceptual & Procedural

Table 1 indicated that the questions prepared by the teachers were classified according to specific types of knowledge. In this classification:

-Questions categorized under procedural knowledge focused on directly calculating the results of mode, median, and arithmetic mean. These questions typically emphasized basic mathematical operations and the application of formulas.

-Questions associated with conceptual knowledge involved examining the relationships among the concepts of mode, median, and arithmetic mean or analyzing the impact of adding or removing data points on these concepts. Such questions aimed to foster a deeper understanding of the underlying concepts.

-Questions containing contextual knowledge were linked to real-life situations and problems. These questions enabled students to connect statistical concepts with their daily lives, facilitating practical understanding.

Additionally, it was observed that some of the questions prepared by teachers integrate multiple types of knowledge simultaneously. In particular, questions combining contextual and conceptual knowledge aim both to enhance students' conceptual understanding and to integrate these concepts into real-life situations.

3. Results

The analysis of mathematics teachers' questions was categorized into subheadings addressing mean, median, and mode-related questions, comparing those formulated before and after the TinkerPlots training.

3.1. Mean-Related Questions Before and After TinkerPlots Training

The examination of mean-related questions in Table 2, designed by teachers prior to TinkerPlots training, indicated a predominant focus on procedural tasks with limited incorporation of contextual or conceptual understanding. These questions were primarily centered on simple calculations, requiring minimal engagement with the data or the underlying concept of the mean. For example, prompts such as "What is the arithmetic mean?" directed students to calculate averages without encouraging deeper reflection on the significance or function of the mean within the data set. This procedural emphasis limits students to superficial application of knowledge, offering little opportunity for critical thinking or connections to real-world contexts.

Table 2. Activity questions prepared by teachers before TinkerPlots training on Mean

Question	Knowledge type
What is the arithmetic mean?	Procedural
Are these measures of central tendency consistent? Why?	Conceptual
What is the arithmetic mean of the data group?	Procedural
Find the average of class 8-D.	Procedural & Contextual
Find the average weight of the class.	Procedural & Contextual
Find the weight average of the class.	Procedural & Contextual
Find the average height of male and female students separately?	Procedural
Find the average for the most consumed product. Explain how you found it.	Contextual & Conceptual
Find the average for the least consumed product. Explain how you found it.	Contextual & Conceptual

Moreover, some pre-training questions incorporated contextual elements by presenting specific data sets or scenarios, yet they predominantly retained a procedural focus. For instance, questions such as "Find the average of class 8-D" and "Find the average weight of the class" introduced context by connecting the calculation to particular groups, such as a class or group of students. While these questions provided a practical application for understanding the mean, they remained centered on computation. They did not prompt students to critically analyze data variability or explore the implications of the mean, thereby limiting opportunities for deeper conceptual engagement.

Following the training, questions encompassed both contextual and conceptual dimensions in addition to procedural tasks, reflecting an advanced understanding of strategies to nurture critical thinking in students. Table 3 revealed a substantial transformation in the nature of questions post-TinkerPlots training, highlighting a significant change in teachers' methodologies for constructing mean-related queries. For instance, questions such as "Which number of students' exit increases the average weight?" and "How will the class average change if Eren and Halil from this class move to another class?" required students to engage in calculations while also examining how modifications to the dataset influence the mean. Such questions facilitated a dynamic and nuanced comprehension of the mean by encouraging students to analyze and interpret data changes, thereby deepening their understanding of central tendency and data distribution.

Table 3. Activity questions prepared by teachers after TinkerPlots training on Mean

Question	Knowledge type
What is the average weight of the students?	Procedural
Which number of students' exit increases the average weight?	Conceptual & Contextual
Which number of students' exit decreases the average height?	Conceptual & Contextual
Find the average number of male and female siblings in the class.	Procedural & Contextual
Find the average income of the students in the class.	Procedural & Contextual
Find the averages of the 1st written and 2nd written separately and comment on the difficulty levels of the 1st and 2nd written.	Conceptual & Procedural
How will the class average change if Eren and Halil from this class move to another class?	Conceptual & Contextual
How will the class average change if Burak, Kerim and Yusuf leave this class?	Conceptual & Contextual
How will the average change if Mehmet, whose grades are 10 and 5 respectively, comes to this class?	Conceptual & Contextual
What is the average age of the students?	Procedural
If which student leaves this group, will the average height increase?	Conceptual & Contextual
If which student leaves, the average weight of the students will decrease?	Conceptual & Contextual
Find and compare the weight averages of male and female students in the class separately?	Conceptual & Contextual
Compare the grade averages of male and female students. Which gender group is more successful?	Conceptual & Contextual
Compare the grade point averages of students with a monthly income above and below 3000₺. Discuss whether financial income affects grade point averages.	Conceptual & Contextual
If 2 students aged 5-10 in class B move to class A, how does the average age change?	Conceptual & Contextual
How will the average age change if 3 students aged 10-15 in class A leave the class?	Conceptual & Contextual

Post-training, questions demonstrated a progression toward tasks integrating procedural calculations with conceptual interpretation. For example, the question “Find the averages of the 1st written and 2nd written separately and comment on the difficulty levels of each exam” not only requires students to compute averages but also to interpret the results in relation to exam difficulty. This approach promoted both procedural proficiency and evaluative reasoning, encouraging students to link statistical measures to meaningful real-world contexts, such as the relative challenges of different exams. Additionally, post-training questions increasingly incorporated comparative and evaluative tasks, urging students to move beyond computation to explore patterns, relationships, and deeper implications. For instance, questions like “Compare the grade averages of male and female students. Which gender group is more successful?” and “Discuss whether financial income affects grade point averages” prompt students to calculate and compare averages while also considering broader social influences on academic outcomes. These tasks enhanced analytical capabilities, fostering critical thinking by situating statistical concepts within social and academic contexts.

Post-training, questions prompted students to evaluate the effect of individual data changes on the mean, fostering deeper conceptual insights. For example, prompts such as “If which student leaves this group, will the average height increase?” and “How will the average change if Mehmet, whose grades are 10 and 5 respectively, joins this class?” moved beyond basic calculations, encouraging students to analyze how specific data adjustments influence the mean. This approach nurtured a comprehensive understanding of the mean as a dynamic and responsive measure, reflecting variations within the dataset rather than serving as a fixed computation. Such tasks might advance students’ ability to adapt their statistical reasoning to diverse scenarios.

In conclusion, the TinkerPlots training significantly enhanced teachers' ability to craft questions that engage students more meaningfully with the concept of the mean. The transition from predominantly procedural to contextually and conceptually rich questions illustrated teachers’ improved capacity to promote critical thinking and analytical skills. By integrating TinkerPlots, teachers developed questions that not only test students’ computational skills but also challenge them to interpret, compare, and evaluate statistical measures in real-world contexts. This pedagogical shift highlights the training's effectiveness in fostering a more nuanced and applicable understanding of the arithmetic mean among students.

3.2. Median-Related Questions Before and After TinkerPlots Training

Table 4 presented the types of median-related questions developed by teachers prior to the TinkerPlots training. An analysis of these pre-training questions indicated that the majority were confined to procedural knowledge, emphasizing the direct computation of the median without encouraging students to engage in contextual or conceptual reasoning. For example, questions such as “What is the median?” and “Find the median!” solely involved basic computational tasks, requiring students to determine the median value. These questions, devoid of real-world context or analytical components, restricted students to applying foundational knowledge with minimal cognitive effort, offering little opportunity to extend their thinking beyond the calculation itself.

Table 4. Activity questions prepared by teachers before TinkerPlots training on the median

Question	Knowledge type
What is the median?	Procedural
Find the median of the grades of the class.	Procedural & Contextual
Calculate the median of the lengths.	Contextual
Find the median of the weights?	Procedural & Contextual
Does the median change when Elif is removed from the group?	Conceptual & Contextual
Find the median for the most sold product.	Conceptual & Contextual

Some of the questions formulated before the training, however, did show contextual elements, attempting to ground the calculation within a specific data set or scenario. For example, questions like “Find the median of the class scores” and “Calculate the median of heights” prompted students to engage with specific data related to class performance or physical attributes, respectively. Though the task remains procedural, adding this contextual layer supported a more realistic application, helping students to connect the procedural calculation with practical scenarios. Nevertheless, even with this added context, these questions still lacked the complexity needed to encourage students to think critically about how or why the median might be relevant in varying data distributions, leaving them with limited conceptual understanding of the topic.

Table 5 illustrated the evolution in question types following the TinkerPlots training, demonstrating significant advancements in teachers’ approaches to designing median-related questions. Post-training, the questions exhibited an increased focus on contextual and conceptual dimensions, reflecting a more integrated approach that promotes both procedural understanding and higher-order thinking. For example, a question such as “What is the median height of the students?” retained the procedural aspect of calculation while embedding it within a realistic context, encouraging students to connect the mathematical concept of median to practical, real-world data. By situating calculations within meaningful scenarios, these questions prompted students to consider the broader significance of the median, thereby fostering deeper engagement and understanding.

Table 5. Activity questions prepared by teachers after TinkerPlots training on median

Question	Knowledge type
What is the median value of the height of the students?	Procedural & Contextual
Calculate the median separately for both written and interpret how there will be a change by looking at questions 1, 2, 3, 4, 5.	Conceptual & Procedural
Find the median. Explain how it was found.	Procedural & Conceptual
How will the median change if 5 students with grades 30, 40, 55, 30, 20 participate from either class?	Conceptual & Contextual

Furthermore, certain post-training questions integrated both conceptual and contextual elements, requiring students to analyze data changes and interpret their impact on the median, thereby fostering a deeper and more nuanced understanding of central tendency measures. For instance, the question, “If five students with scores of 30, 40, 55, 30, and 20 join the class, how does the median change?” not only involved performing calculations but also prompted students to consider the implications of incorporating specific values into the dataset. This approach might encourage critical thinking, as students must reflect on how variations within the dataset influence the median, positioning it as more than a mere computational step. Similarly, the question, “Calculate the median for both exams separately and interpret any differences according to questions 1, 2, 3, 4, and 5,” challenged students to go beyond basic computation by interpreting the relative difficulty of each exam based on the median. This task connected statistical analysis with evaluative reasoning, encouraging students to link mathematical concepts to meaningful comparisons and insights.

Another significant addition to the post-training questions was the incorporation of tasks that combined procedural calculations with conceptual explanations. For example, the question, “Find the median and explain how you found it” required students to not only compute the median but also articulated their process, promoting metacognitive reflection on their methodology. This type of question might encourage deeper engagement with the concept, as students must analyze and justify their reasoning, fostering critical and flexible application of statistical concepts.

Overall, the findings suggested that the TinkerPlots training had a substantial impact on teachers' question formulation strategies, shifting their focus from purely procedural tasks to more sophisticated, contextual, and conceptual inquiries. This progression reflected an enhanced capacity among teachers to design questions that stimulated students' analytical thinking and deepen their understanding of the median. By integrating TinkerPlots, teachers adopted a more nuanced questioning approach, balancing procedural competence with the ability to apply statistical measures meaningfully and critically in real-world scenarios. This shift underscored the effectiveness of TinkerPlots in improving instructional strategies, enabling teachers to support a more comprehensive and profound understanding of statistical concepts in their students.

3.3. Mode-Related Questions Before and After TinkerPlots Training

The analysis of questions, based on teachers' preparations before the training as seen in Table 6, indicated that the majority were confined to a procedural level. These questions were primarily focused on identifying the mode, without encouraging students to engage in contextual or conceptual reasoning. For example, questions like "What is the mode?" or "Find the mode!" required students to perform a single operational task to determine the mode value. Such questions lacked depth, offering no opportunity for contextual application or conceptual exploration, and remained limited to basic knowledge application.

Certain pre-training questions demonstrated contextual elements. For instance, prompts such as "Find the mode of the Math scores for class 8-D!" and "What is the mode of ages?" incorporated context by referencing specific datasets or real-world scenarios. Nevertheless, these questions predominantly emphasized procedural execution, requiring minimal conceptual analysis or deeper understanding from the students.

Table 6. Activity questions prepared by teachers before TinkerPlots training on mode

Question	Knowledge type
What is the mode?	Procedural
Find the mode of Math scores of class 8-D.	Procedural & Contextual
Find the mode of age.	Procedural & Contextual
Calculate the mode of the lengths.	Procedural & Contextual
Find the mode for the data. Explain how you found it.	Conceptual & Procedural

The post-training question stems, as illustrated in Table 7, exhibited a marked enhancement in both contextual and conceptual elements, reflecting a substantial improvement in teachers' question-framing strategies. For example, prompts such as "What is the mode of the number of siblings?" and "What is the mode of heights?" integrate specific contexts that encourage students to relate the task to real-world scenarios. These questions go beyond requiring procedural calculations, prompting students to interpret the data and reflect on its relevance to the posed question.

Table 7. Activity questions prepared by teachers after TinkerPlots training on Mod

Question	Knowledge type
Find the mode of the number of siblings.	Procedural & Contextual
What is the mode of their height?	Procedural & Contextual
Examine the change in the mean, mode and median of the data when Sergen leaves this class?	Conceptual & Contextual
What is the mode of the data set by looking at the grades? How does the mode change if 4 students with a grade of 30 join?	Conceptual & Contextual
Find the modes of weight and height.	Procedural & Contextual
Calculate the mode separately for both writings and interpret according to questions 1, 2, 3, 4, and 5.	Conceptual & Procedural

Among the post-training questions, some effectively combined contextual and conceptual characteristics. For instance, the question “How would removing Sergen from the class affect the mean, mode, and median?” encouraged students to analyze the impact of changes in the data set on measures of central tendency. This question required a conceptual understanding of how data variability influences statistical measures, fostering deeper engagement with the underlying concepts. Similarly, the question “What is the mode of the data set? If four students with a score of 30 are added, how does the mode change?” offered students the opportunity to think about and analyze how changes in the data set impact the concept of mode. Such questions fostered students' conceptual thinking skills. Additionally, post-training questions revealed that teachers started to blend procedural and conceptual elements in their questions. For instance, “Calculate the mode for both exams separately and comment on each according to questions 1, 2, 3, 4, and 5” required students first to complete an operation and then to interpret the results in relation to other questions. This type of question supported the development of both procedural skills and conceptual understanding.

In conclusion, the post-training questions reflected a significant shift toward integrating contextual and conceptual elements, emphasizing deeper engagement with statistical concepts. By incorporating questions that required students to analyze the effects of changes in data sets on measures of central tendency, teachers fostered critical thinking and conceptual understanding. Furthermore, the blending of procedural tasks with interpretative components demonstrated a balanced approach that supported the simultaneous development of computational skills and higher-order reasoning. This progression highlighted the positive impact of the training on teachers' ability to design questions that promoted both procedural fluency and conceptual depth in their students.

4. Discussion

This study examined the types of mathematical knowledge reflected in mathematics teachers' questions about the concepts of mean, median, and mode during TinkerPlots training. The findings highlighted the impact of dynamic software such as TinkerPlots on enhancing contextual, conceptual, and procedural knowledge, particularly in the questions crafted by teachers. TinkerPlots training enabled mathematics teachers to design questions encompassing richer and more diverse types of knowledge, fostering a deeper understanding of statistical concepts.

The findings revealed that prior to TinkerPlots training, teachers predominantly focused on procedural knowledge in their questions. For instance, questions such as "What is the arithmetic mean?" or "Find the median of this data set" illustrate that teachers tended to create questions that merely required following specific procedures without offering conceptual or contextual depth. This finding suggested that teachers' pedagogical practices remained at a superficial level, limiting students' opportunities to establish meaningful connections with statistical concepts (Biggs & Tang, 2011). Following the TinkerPlots training, however, a significant transformation was observed in teachers' questions. Questions were restructured to include not only procedural knowledge but also conceptual and contextual knowledge. For example, questions such as "How does the class average change when Eren and Halil leave the classroom?" or "Compare and interpret the average grades of students with an income level above 3000₺ and those below" demonstrate that teachers moved away from a pedagogy that solely followed procedures. Instead, they began formulating more complex questions aimed at analyzing the structure of data and interpreting changes within that structure. This transition allowed mathematics teachers to transcend superficial teaching methods, promoting a more profound comprehension of the evolving nature of statistical concepts and seamlessly embedding this insight into their pedagogical approaches.

Another significant finding of the study was the enrichment and diversification of contextual knowledge-based questions through TinkerPlots training. Mathematics teachers gained the opportunity to move beyond abstract calculations and instead support the interpretation of questions within meaningful contexts (Saxe, 1988). Through this approach, mathematics teachers were able to create a meaningful and interactive learning environment for students (Freudenthal, 1991; Sáenz, 2009), contributing to a deeper understanding of statistical concepts by linking them to real-life situations. This demonstrated that teachers not only prioritized conceptual and procedural knowledge but also placed significant emphasis on contextual knowledge. It highlighted their intention to foster students' statistical reasoning skills by connecting these skills to everyday life, thereby promoting a more comprehensive and applied understanding of statistical concepts.

Prior to the TinkerPlots training, mathematics teachers predominantly posed questions centered on routine mathematical calculations. The training, however, enhanced their ability to visualize, analyze, and interpret statistical concepts. TinkerPlots, in particular, equipped teachers with interactive and visual analytical tools, enabling a deeper understanding of data's dynamic nature. These capabilities were instrumental in incorporating contextual and conceptual elements into the design of their questions. Moreover, the findings indicated that TinkerPlots enabled teachers to move beyond the understanding and application levels of Anderson and Krathwohl's (2001) taxonomy of knowledge, advancing to the analysis and evaluation levels. For instance, post-training questions such as, "How do measures of central tendency change when certain students leave or new students join the class?" illustrated a pedagogical shift from merely teaching procedural calculations to adopting a deeper instructional approach. This approach emphasized data manipulations and the interpretation of their effects, fostering a richer and more meaningful learning experience for students.

This study highlighted the significance of TinkerPlots training in shaping teachers' approaches to statistical thinking and, consequently, its importance in enhancing their professional competencies. Before the training, teachers primarily focused on procedural knowledge; however, post-training, they developed the ability to formulate more complex and multidimensional questions, thereby advancing their pedagogical strategies. In this context, Shulman's (1987) concept of pedagogical content knowledge emphasizes the need for teachers to integrate content knowledge with pedagogical strategies to create effective teaching practices. Through TinkerPlots training, teachers had the opportunity to achieve a profound transformation in their pedagogical approaches and make meaningful contributions to their professional development. The questions formulated by teachers after the training particularly highlighted the dynamic nature of statistical concepts and the significance of understanding changes within these structures. For instance, questions such as "What is the impact of added or removed values in a data set on measures of central tendency?" demonstrated the development of deeper pedagogical insights among teachers. This transformation demonstrated that, through the professional development process, teachers not only deepened their content knowledge but also developed the skills to apply this knowledge in their teaching practices, highlighting the dual benefits of the training in enhancing both subject mastery and instructional proficiency.

This study also underscored the central role of professional development programs like TinkerPlots in transforming teachers' pedagogical skills. Recent research has highlighted that such programs significantly enhance teachers' ability to incorporate conceptual and contextual dimensions into their teaching practices, thereby fostering deeper student engagement and understanding (Postholm, 2018; Darling-Hammond, 2017). These programs enable mathematics teachers to approach mathematical knowledge types in a more enriched manner and integrate this knowledge into their practices more effectively. For example, studies by Groth et al. (2019) and Desimone and Garet (2015) show that professional development activities focusing on technology-enhanced learning tools empower teachers to adopt innovative pedagogical strategies, leading to improved student outcomes. Furthermore, Guskey (2020) emphasizes the importance of designing these programs to include collaborative opportunities, allowing teachers to exchange ideas and reflect on their teaching methods.

In developing professional development programs, the priority should not only be to provide teachers with subject-specific knowledge but also to equip them with strategies to teach this knowledge through conceptual and contextual dimensions. Such an emphasis could make teachers' pedagogical practices more comprehensive and impactful. Recent studies have shown that when teachers are trained to use tools like TinkerPlots effectively, their ability to create meaningful learning experiences increases significantly, ultimately benefiting both teachers and students (Lajoie, 2018; Heid et al., 2019).

5. Conclusions

In conclusion, TinkerPlots training has deepened teachers' pedagogical approaches and fostered a lasting transformation in their professional development processes. The study underscored the importance of professional development programs such as TinkerPlots in advancing teachers' instructional methodologies. The results highlighted a significant improvement in teachers' ability to formulate questions, enabling them to engage with statistical concepts in a more profound and contextually relevant manner. Professional development programs should focus not only on increasing knowledge levels but also on building the capacity to effectively apply this knowledge in instructional practices. Tools such as TinkerPlots offer substantial support for mathematics teachers in their professional development, fostering the adoption of more effective and contemporary instructional strategies. Future studies could explore how mathematics teachers utilize such dynamic software in their lessons and the role it plays in developing students' statistical thinking skills. Since this study did not include teachers' in-class practices, subsequent research could investigate how such diverse questions are implemented in classrooms and how they contribute to enhancing students' statistical thinking abilities. Additionally, how contextual, conceptual, and procedural questions enhance statistical thinking and how students respond to these questions can be examined.

Declaration of Conflicting Interests and Ethics

The authors declare no conflict of interest.

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