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3D geometric thinking skills of preschool children

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Abstract

Early childhood mathematics education takes attention for decades. The effect of qualified mathematics education during early years has an impact on children's future academic success. Main purposes of the study were twofold; to examine children's current 3D geometry thinking skills and to investigate the development of children's 3D geometry thinking skills after a 3D training program developed by researchers, and named as 3D in Early Childhood (3DinEC). The qualitative research design with convenience sampling method were utilized. The participant children were three girls and four boys with the age range of 60 months to 72 months. The data were collected using a semi-structured interview form including six abilities (Pittalis and Christou, 2010), and 15 items. In the analysis of the data, descriptive and content analysis methods were used. Findings indicated that participant children had a limited understanding regarding 3D geometric thinking. It was also found out that some of these thinking skills like identification of 3D geometric shapes and recognition of these shapes' properties, etc. could be enhanced through various activities.

Keywords: 3D training, geometric thinking, preschool education, preschool children

1. Introduction

The quality of early childhood mathematics education is a significant question and takes global attention for decades. Early childhood experiences have an influence and long term effects on children's future learning and also on achievement in mathematics, other subjects as well as in real life (Adelman, 2006; Duncan et al., 2007; Ginsburg et al., 2008; Jordan, Kaplan, Ramineni, and Locuniak, 2009; Reyna, Nelson, Han, and Dieckmann, 2009). Geometry as a mathematical content taught in early childhood curriculum is significant for children to understand real world. However, geometry and spatial thinking do not play an important role in the practice of early childhood mathematics education (Rittle-Johnson, Zippert, and Boice, 2018; Sarama and Clements, 2009b; Davis and The Spatial Reasoning Study Group, 2015).

Geometry begins with play and playful activities like using mosaics, paper folding, drawing, and pattern blocks can enrich visual structures of children and can enhance their knowledge about shapes and their attributes (van Hiele, 1999). For instance, there

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are various studies how playing with blocks affects preschool children's development of understanding of geometry (Casey et al., 2008; Caldera et al., 1999; Ferrara et al., 2011; Ramani et al., 2014). Besides, shapes are fundamental constructs in and beyond geometry. And children's understanding of shapes begins in early years. There are also studies about how children perceived shapes (Aktaş-Arnas and Aslan, 2004; Clements et al., 2019; Clements et al., 1999; Hannibal and Clements, 2000; Satlow and Newcombe, 1998). Moreover, there is growing body of knowledge about spatial skills and the significance of and relation with mathematics (Levine, Ratliff, Huttenlocher, and Cannon, 2012; Newscombe, 2010; Verdine, Golinkoff, Hirsh-Pasek, and Newscombe, 2014; Uttal et al., 2013). Spatial skills connects math to physical world and these skills links with early mathematics achievement.

Spatial abilities are seen as predictors of educational success. There are various conceptions of spatial ability. Generally, it is defined as skills involving the retrieval, retention and transformation of visual information in spatial context (Halpern, 2000). There are specific factors that constitute spatial ability which are spatial orientation, spatial location memory, targeting, spatial visualization, disembedding and spatial perception (Kimura, 1999). Clements (1998), on the other hand, subdivided spatial ability into two factors as spatial visualization and spatial orientation. Lohman's (1988) influential model, as considered in the present study, supported the existence of three factors which were spatial visualization, spatial orientation, and spatial relations. First of all, spatial orientation is about accurately estimating orientation changes, then, spatial visualization is generally defined as ability to recognizing and quantifying the orientation changes. Spatial visualization is named as mind's eye and defined also as "a specific type of spatial thinking that involves using our imagination to generate, retain, retrieve, and transform well-structured visual images" (Lohman, 1996, p.98). Lastly, spatial relation is about rotating a spatial object as a whole fast and correctly (Colom et al., 2001).

There are difficulties concerned with visualization and imaginary (Dreyfus, 1991; Love, 1995). These difficulties arise while constructing, re-presenting, and transforming. Love (1995), for instance, mentioned that in geometry the relationship between mental object and physical image is especially the difficult one. The role of visualization is also stressed in geometrical thinking (Presmeg, 2006). Geometrical thinking includes defining, understanding and classifying geometrical shapes, understanding the relationship between geometrical shapes, reasoning, making proof, visualization and drawing. According to National Council of Teachers of Mathematics (NCTM, 2000), 3D geometrical abilities are constructing nets, representing 3D solids by 2D figures, identifying solids and their elements, structuring arrays of cubes, calculating surface area and volume of solids, and comparing the attributes of solids. These abilities mentioned in NCTM as well as abilities considered by Pittalis and Christou's (2010) 3D geometry thinking test were taken into account in the present study. In the present study, we examined spatial abilities specifically abilities that are closely related with 3D geometry. Main purposes of this study are twofold: (1) to examine children's current 3D geometry thinking skills; and (2) to investigate the development of children's 3D geometry thinking skills after a 3D training program developed by researchers and named as 3D in Early Childhood (3DinEC).

1.1. Theoretical Background

1.1.2. 3D geometric thinking and spatial abilities

Geometrical thinking provides an environment for defining, understanding and classifying geometric objects, understanding geometric relations, developing new arguments, reasoning, making proofs, visualizing, and drawing geometrical shapes and objects (NCTM, 2000). Van Hiele's geometric thinking levels are age independent and hierarchical. Therefore, a child in a certain age can understand and think better than a child who is older than himself. As well, van Hiele (1999) emphasized that geometry that will be taught to children should be appropriate to children's geometrical thinking level, clearly, appropriate to children's readiness as also mentioned by Piaget. Van Hiele also suggested considering appropriate geometrical experiences.

Actually, we live in a three dimensional world but there are two representations used for 2D and 3D geometrical objects; drawing and model (Parzysz, 1988). 2D objects' representations are generally done with drawings while 3D objects' representations are done both with models and drawings. In lower grades some teachers prefer using concrete models like sticks for representing 2D shapes. However, this could cause problems like not being able to visualize the properties of shapes. Otherwise, 3D objects' representations are generally done with models. Drawing for 2D shapes and models for 3D objects are named as close representations, and they give more information about themselves. However, distant representations like models for 2D shapes and drawings for 3D objects are also significant for children to use their cognitive abilities. There are now three representations used for 3D geometrical objects; models, animations on pcs, and drawings (Gutierrez, 1992). These representations are significant for various reasons and their usage had both advantages and disadvantages.

Including 3D geometry in the mathematics curriculum provides opportunities for students to develop spatial awareness, intuition in geometry, visualization, knowledge, understanding, usage of geometrical properties and theorems (Jones, 2002). Duval (1998) suggested that geometrical reasoning involves cognitive processes like visualization, construction, and reasoning processes. The synergy between these processes is necessary for achieving in geometry. While teaching 3D geometric thinking, promoting the development of students' spatial abilities is generally underlined. Especially when teaching and learning 3D geometry is concerned, spatial visualization as a spatial ability influences students' achievement in it (Gutierrez, 1992). That is to say, children's understanding about 3D geometry is associated with spatial abilities especially with visualization. To Couto and Vale (2014), geometry is described as comprising of visualization and comprehension of shapes. Clements (1982) defined visualization as a significant aspect of spatial reasoning. Geometry is made up of spatial orientation and spatial visualization (Van Klinken, 2010). Moreover, Yeh (2013) mentioned about three types of spatial abilities; spatial visualization, spatial orientation, and spatial relations. Specifically, visualization is not just seeing and describing an object, instead, it is an ability to represent, transform, generate, document, and reflect on visual information (Hershkowitz, Ben-Chaim, Hoyles, Lappan, Mitchelmore, & Vinner, 1990).

Gutierrez (1992) studied the relationship between 3D geometrical thinking and van Hiele's geometrical thinking and developed four levels for 3D geometric thinking. In the first level (recognition) children perceive solids as a whole and do not consider their components. But still they had an idea about these components like angle, size, edge, length, and parallelism. Second level (analysis) involves processes like identifying attributes like the number of faces, their shapes, and the number of vertices. But still children do not perceive the relationship between attributes. In the third level (informal deduction) children can classify solids based on their attributes and definitions are meaningful to them. Last level (deduction) comprises proving theorems about 3D geometry. About geometric thinking, Pittalis and Christou (2010) developed five abilities. These abilities are manipulating different representational modes of 3D objects, recognizing and constructing nets, structuring 3D arrays of cubes, recognizing 3D shapes, their properties and comparing them, and calculating the volume and area of solids. In the present study, these abilities about 3D geometric thinking were considered as a guideline.

There are other issues reading 3D geometric thinking, such as drawing, area and volume. First of all, drawing a 3D object should be focused on as well should be interpreted (Pittalis and Christou, 2010). To Deregowski and Bentley (1987), for understanding the drawing of 3D object, the depth of drawing, and the elements of it, individuals should be visualized it in mind. Murphy and Wood (1981) mentioned that 4 years old children could benefit from their knowledge about painting in geometric drawing tests. About area there are two interesting studies; unit squares were easy to use for taking space and during these processes experiences played a significant role (Heraud, 1987; Owens and Outhred, 2006). Lastly, counting unit cubes in a structure could be useful for understanding the idea of volume (Battista and Clements, 1996). This idea would help them to construct volume formula of the structure. There are other studies how students perceived shapes, visualization, geometric thinking, and specifically 3D geometric thinking. For instance, in one of the studies about geometric shapes Nieuwoudt and van Niekerk (1997) found out that first graders often named cubes as squares. Moreover, Charalambos (1997) analyzed basic characteristics of geometrical shapes and tried to describe how students learn basic geometrical concepts. As a result, he found out that shapes play an essential role and students' errors were generally because of prototype phenomenon. About 3D thinking, plan representations are generally used for representing 3D geometrical objects at school and during teaching processes these representations are often found to be taken into consideration as real objects (Berthelot and Salin, 1998). Another significant study was done by Denizli and Erdoğan (2018), based on Pittalis and Christou's abilities defined, they prepared a test for determining 1st to 4th grade students' 3D geometrical thinking skills. They found that students' 3D geometrical thinking skills were increased as grades got higher.

2. Method

In the study, it was aimed to understand children's 3D geometry thinking skills and the development of children's 3D geometry thinking skills after 3D in Early Childhood (3DinEC) training program, the present study conducted in Agri, east of Turkey. The study adopted the qualitative approach, which tried to interpret meaning out of the collected data, and helped understanding social issues based on the participant or other data sources.

2.1. Participants

Participants of the study were seven children (3 girls, 4 boys) varied from 60 months to 72 months of age in a public preschool selected via convenient sampling method, so that it became economical and wasn't time consuming (Yıldırım and Şimşek, 2006). Participation of the children was arranged upon obtaining their parents' written consent, and the children's declaring their willingness. The ethical approval was issued by the Ethical Board of Agri University, and the research permission in the schools from Agri Directorate of National Education. The age range of participant children.

2.2. Data collection tools

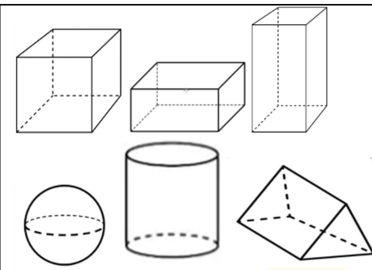
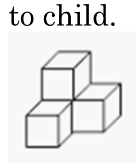
To determine the children's 3D geometric thinking skill, a semi-structured interview form was used. This data gathering tool included six abilities determined by Pittalis and Christou (2010). Before using this tool, it was submitted to two experts in the field of mathematics education and also early childhood education for their opinions. In the interview form there were totally 15 questions. Each of them also had sub-questions. Just after receiving their feedbacks, necessary changes were conducted in the form. These changes were mostly related to wordings.

The pilot interviews were done with two six-year-old children. In the light of pilot interviews, the order of questions was changed to take the attention of children in the process. The school administration provided a quiet room at the school for the actual study.

This tool included tasks; four of them were prepared to do on paper and other tasks were supposed to do with unit squares, unit cubes, empty box, 3D geometric shapes like rectangular prism, cube, square prism, triangular prism, cylinder, and sphere, 2D geometric shapes like square, triangle, circle, and rectangle, openings of 3D shapes.

Table 1
3D geometric thinking test

Ability	Definition of Tasks	Example
Recognition and construction of nets	1. Identification of 2D representation of 3D geometric shapes 2. Construction of 3D shapes with 2D shapes	* Show 3D cube, rectangular prism, triangular prism, square prism, cylinder to child one by one. Then ask child to match their 2D nets.

	3. Identification of 3D shapes 4. Identification of openings of 3D shapes	 Ask children reasons behind his/her choice. Similarities and differences?
Manipulation of 3D shapes representation modes	1. Translation of 3D geometric shapes into isometric 2. Translation of isometric side, top, and front projection views of 3D shapes in 3D objects. 3. Recognition of cube drawn into an isometric view.	* Give child 3D cube, rectangular prism, triangular prism, square prism, cylinder one by one. Ask him/her to analyze each. Then, ask child to draw each geometrical shape's picture on the given paper. Ask child which factors s/he consider while s/he is drawing.
Structuring 3D arrays of cubes	1. Enumeration of the cubes needed to transform an object into prisms. 2. Enumeration of the cubes that fit in a not empty box. 3. Enumeration of the cubes that fit in an empty box.	* Show and illustrate following incomplete 3D figure composed of cubes to child.  Then, show 3D rectangular prism and square prism to child. Ask child to complete given incomplete figure to make it resemble a rectangular and a square prism one by one. Ask child to guess how many cubes s/he needs for completing the figure into a rectangular prism or a square prism. Give child a chance to try completing the figure. Ask the difference of cubes s/he used and his/her guess. Ask his/her reasons behind his/her choices.
Recognition of 3D shapes' properties	1. Recognition of 3D shapes. 2. Enumerating the vertices, faces, and edges of 3D shapes.	* Show 3D cube, rectangular prism, triangular prism, square prism, cylinder to child one by one. Ask him/her to analyze each. Show child what is corner, surface, edge of a random geometrical shape.

		Then, show child each geometrical shape and ask him / her how many edges, corners, surfaces it has and names of surfaces as well. Ask child to show its edge, corner and surfaces as well.
Calculation of the volume and the area of solids	1. Calculation of the area of 3D shapes with unit squares 2. Calculation of the area and volume of 3D geometric shapes presented as open nets 3. Calculation of the volume of 3D shapes with unit cubes 4. Comparing the area and the volume of 3D shapes	* Give child cube, rectangular prism, and square prism one by one. Then, give child unit squares as much as possible. Ask child to guess how many unit squares s/he needs for covering its all surfaces. Then, give child a chance for trying her /his guess. Then, ask the reasons between his /her guess and actual result. * Give child cube, rectangular prism, and square prism one by one. Then, give child unit cubes as much as possible. Ask child to guess how many unit cubes s/he needs for filling its volume. Then, give child a chance for trying her /his guess. Then, ask the reasons between his /her guess and actual result.
Comparison of 3D shapes properties	1. Right/ wrong answers referring to the elements and properties of 3D shapes	* Following questions are asked to children if these eight statements are true or false. Children are given 3D geometric shapes while they are answering these statements. Cube is composed of squares. Square prism is just composed of rectangles. A sphere does not have any edge or corner.

2.3. Data Collection Process

Necessary ethical procedures were approved by Ethical Board of the University and as well by Ministry of National Education. Children's parents were informed about research purposes and research process. Then, parents were asked to give written signed consents for their children to participate to data gathering process and 3DinEC training program. Besides, children's willingness was asked just before data gathering processes as well as each session of 3DinEC training program. They all expressed their willingness to participate data gathering processes and 3DinEC training program.

Clinical interviews included initial standardization of the task, use of objects around which the task revolves, a how and why questions, an immediate interpretation of the subject's response, on-the spot hypothesis making and testing, the freedom of improvise, and evolution of the interview process over a period of time (Ginsburg, 1997). The study was conducted accordingly. Clinical interviews were done before and after 3DinEC training program. Each student was interviewed one by one. Each interview took about 20 minutes to 45 minutes.

2.4. 3DinEC Training Program

3DinEC Training program and its activities were developed by the researchers. It was controlled by two other academicians who had experiences in early childhood education and mathematics education. Seven hours of training program were planned and applied within a classroom through two weeks. Videos as well as photographs were collected during trainings.

3DinEC included six main geometric shapes; cube, cylinder, triangular prism, rectangular prism, square prism, and sphere. Each geometric shape was introduced one by one. Activities done were as follows;

- Constituting geometric shapes with 2D shapes,
- Constituting and determining geometric shapes from the nets of 3D shapes,
- Colouring the determinant face of 3D geometric shapes,
- Discovering edges, sides, and faces by using 3D objects and numbering these characteristics.
- Likening geometric shapes to the real life objects or anything they had seen before,
- Telling stories about each geometric shapes or developing new stories with children,
- Singing and developing songs about 3D geometric shapes,
- Asking riddles,
- Using drama,
- Playing in 3D shapes,
- Presenting real objects, they had taken from their houses,
- Using animations for visualizing 3D geometric shapes altogether.

During application of activities, children were first introduced with geometric shapes. One of the researchers applied activities at school through 3DinEC training. During activities, researcher gave children chance to examine geometric shapes individually or in groups. They were generally encouraged to talk about similar shapes they had seen in their real lives or they just played with these shapes. Then, the researcher introduced the activity of the day one by one. The researcher and children made the activities together. In some of the activities, the researcher asked children to talk about their parents about these shapes or took objects similar to geometric shapes to 3DinEC training.

2.5. Data Analysis

The data gathered were analysed through descriptive and content analysis methods. Video records were transcribed. Children's reasoning for their answers to each task were coded. Then, related codes were constituted under themes. For reliability of coding and constituting theme processes two researchers studied the transcribed data separately. Their codes were compared with Miles and Huberman's (1994) inter-coder reliability formula, and it was found to be 96 % codes. Then, themes were sent to another researcher to control. According to his comments, codes determined were found to be appropriate to criteria presented on the rubric.

A rubric was also prepared for classifying children's answers to each question. According to rubric, task under each ability was classified as; right, partially right, or wrong.

3. Results

Children's 3D geometric thinking skills were compared as before 3DinEc and after 3DinEc one by one along with their expressions through training. Abilities are presented one by one in the following tables.

Table 2

Recognition and construction of nets ability before and after 3DinEc training

Ability	Category	Before 3DinEC Training			After 3DinEC Training		
		R*	PR**	W***	R*	PR**	W***
Recognition and construction of nets	1. Identification of 2D representation of 3D geometric shapes	6	1	-	7	-	-
	2. Construction of 3D shapes with 2D shapes	-	2	5	5	2	-
	3. Identification of 3D shapes	-	1	6	6	1	-
	4. Identification of openings of 3D shapes	4	3	-	7	-	-

*Right, **Partially right, ***Wrong

As seen in Table 2, children could identify 2D representation of 3D geometric shapes and distinguish some of the openings of 3D shapes before 3DinEC training. However, they could not construct 3D shapes with 2D shapes and identify 3D shapes. After 3DinEc training they could easily identify 2D representation of 3D shapes and openings of 3D shapes. Besides, children could generally identify 3D shapes and construct 3D shapes with 2D shapes.

For identifying 2D representations of 3D geometric shapes, children had given chance to express their reasons behind their choices. They were generally concentrating on determinant faces of geometric shapes. All of them mentioned that determinant faces of both 2D and 3D shapes for their choices. Except one child could not identify rectangular prism from square prism which were presented on 2D paper. After 3DinEC training, all children could achieve first category. While they were expressing their reasons behind their choices. They concentrated on all faces of geometric shapes as well as to determinant faces.

About second category, before 3DinEC training participant children had difficulties regarding constructing 3D shapes with 2D shapes. For instance, only one child could identify triangular prism with its all faces and the other child could identify cylinder without making false. These two children could not construct other geometric shapes with 2D shapes. However, after the training children generally could construct 3D geometric shapes with the help of 2D shapes. Two children had problems with rectangular prism and sphere.

The other category is identification of 3D shapes. Children had also problems in this category and they identified these shapes with 2D shapes' properties before 3DinEC training. They stated that they learned them from their parents or their teacher. Only one child stated he learned them from his elder brother. However, after the 3DinEC training almost all children could identify these shapes and only one child could not identify rectangular prism. They all mentioned they learned them from the training.

About identification of openings of 3D shapes, some of the children could identify 3D shapes' openings. They also mentioned that determinant faces helped them to match with 3D shapes. Before the training, three children had problems especially with square prism and cube. But after the training all of the children could identify these shapes' openings and they could express the significance of determinant faces of geometric shapes.

Table 3

Manipulation of 3D shapes representation modes ability before and after 3DinEc training

Ability	Category	Before 3DinEC Training			After 3DinEC Training		
		R*	PR**	W***	R*	PR**	W***
Manipulation of 3D shapes representation modes	1. Translation of 3D geometric shapes into isometric	-	1	6	1	5	1
	2. Translation of isometric side, top, and front projection views of 3D shapes in 3D objects.	-	4	3	2	4	1
	3. Recognition of cube drawn into an isometric view.	-	4	3	3	3	1

*Right, **Partially right, ***Wrong

About manipulation of 3D shapes representation modes ability, children had difficulties regarding all categories under this ability. Even after training children had problems in these categories. Few children could achieve these categories after training and participant generally could do these categories partially right.

Before 3DinEC training, children could not translate 3D geometric shapes into isometric. Only one child can translate cylinder into isometric. After the training only few children could translate shapes into isometric. Most of children could only translate cylinder and sphere into isometric. Second category is about translation of isometric views into 3D objects. Under this category, children could not achieve this category before the training but after the training two children could do this category and others generally could do partially right. Recognition of cube drawn into an isometric view is the last category, and under this category, children had problems as well before the training. After the training, three children could achieve this category.

Table 4
Structuring 3D arrays of cubes ability before and after 3DinEc training

Ability	Category	Before 3DinEC Training			After 3DinEC Training		
	.	R*	PR**	W***	R*	PR**	W***
Structuring 3D arrays of cubes	1. Enumeration of the cubes needed to transform an object into prisms.	-	1	6	3	4	-
	2. Enumeration of the cubes that fit in a not empty box.	3	NA****	4	5	NA****	2
	3. Enumeration of the cubes that fit in an empty box.	1	NA****	6	3	NA****	4

*Right, **Partially right, ***Wrong ****Not Applicable

Table 4 presents findings regarding structuring 3D arrays of cubes ability. Before 3DinEC training children could not enumerate the cubes needed to transform an object into prisms category, but only three could achieve this category after the training. About enumeration of cubes that fit in an non-empty box and an empty box categories, children's achievements had increased after 3DinEC training especially children could enumerate the cubes that fit in a not empty box. While they were asked to estimate before trying, they generally preferred counting by finger. They preferred this method both before and after training.

Table 5
Recognition of 3D shapes' properties ability before and after 3DinEc training

Ability	Category	Before 3DinEC Training			After 3DinEC Training		
		R*	PR**	W***	R*	PR**	W***
Recognition of 3D shapes' properties	1. Recognition of 3D shapes.	-	1	6	6	1	-
	2. Enumerating the vertices, faces, and edges of 3D shapes.	1	1	5	5	2	-

*Right, **Partially right, ***Wrong

Under this ability as seen in Table 5, there have been improvements in children's answers. Children could easily recognize 3D shapes and could enumerate these shapes' vertices, faces, and edges. Only two children had problems and could answer partially rightly. Findings indicated that there has been an increase in children's recognition of 3D shapes' properties.

Table 6
Calculation of the volume and the area of solids ability before and after 3DinEc training

Ability	Category	Before 3DinEC Training			After 3DinEC Training		
		R*	PR**	W***	R*	PR**	W***
Calculation of the volume and the area of solids	1. Calculation of the area of 3D shapes with unit squares	-	-	7	-	2	5
	2. Calculation of the area and volume of 3D geometric shapes presented as open nets	-	-	7	-	3	4
	3. Calculation of the volume of 3D shapes with unit cubes	-	-	7	1	5	1
	4. Comparing the area and the volume of 3D shapes	2	3	2	5	2	-

*Right, **Partially right, ***Wrong

Children had most difficulty in calculation of the volume and the area of solids ability. Except comparing the area and volume of 3D shapes category, there was not any significant difference in other categories found under this ability even after the training. For instance, only one child could calculate the volume of 3D shapes with unit cubes after the training. He could calculate the volume of cube with unit cubes. Some children could achieve these categories partially right in some geometric shapes (square or rectangular prism's volume along with cube' volume). In the last category, children had shown success in comparing volume or area of 3D shapes. Two children answered partially right, these children could make comparisons in volumes but not in areas of 3D shapes.

Table 7
Comparison of 3D shapes properties ability before and after 3DinEc training

Ability	Category	Before 3DinEC Training			After 3DinEC Training		
		R*	PR**	W***	R*	PR**	W***
Comparison of 3D shapes properties	1. Right/ wrong answers referring to the elements and properties of 3D shapes		3	4	6	1	-

*Right, **Partially right, ***Wrong

Comparison of 3D shapes properties was the ability investigated. After the training, children's answers were improved significantly. Children generally could answer the questions referring to the elements and properties of 3D shapes rightly. Before the training, children had problems in comparing 3D shapes' properties, however, after the training almost all children could do this category. There has been an increase in children's right answers. While they were answering right/wrong questions, they used 3D shapes for showing the researcher they were right.

4. Discussion and Conclusion

3D geometric thinking in early childhood is a complex area, and a need for more research on 3D geometrical thinking in early childhood education has become clear. Developing spatial awareness, intuition in geometry, visualization, knowledge, understanding usage of geometrical properties and theorems could be achieved through 3D geometry (Jones, 2002). The present study may address how 3D geometrical thinking can be enhanced and understood in early childhood practice. Therefore, the present study examined children's 3D geometry thinking skills and also investigated the development of children's 3D geometry thinking skills after 3DinEC. There are both conflicting and approving issues with current literature. The results revealed that children had a limited understanding about 3D geometric shapes.

According to NCTM (2000), children between preschool to 2nd grade level could distinguish, name, build, compare 3D objects, explain 3D objects' properties and parts, investigate and make guesses about what will happen if parts come together or separate. Moreover, they could define related states in space, comment on direction and distance by using simple words like "next to," realize movements such as rotating-turning-scrolling, symmetrical recognizing, and creating shapes, and as well with spatial visualization objects they could animate in mind, recognize and represent objects from different perspectives, associate numbers and measurement with geometry (NCTM, 2000). Findings of the study confirm that participant children could achieve some of these competencies especially after 3DinEC training and could not achieve the others even after the training. These competencies will be discussed in the following paragraphs.

First of all, findings indicated that preschool children were successful in identification of both 2D representations of 3D geometric shapes and openings of 3D shapes. Especially determinant faces of 3D geometric shapes were helpful. Similar results were found in Nieuwoudt and van Niekerk's (1997) and Charalambos' (1997) studies and they generally stated that basic characteristics of shapes played a significant role in determination of them. The present study also indicated that children had problems regarding construction of 3D shapes with 2D ones and identification of 3D shapes categories but the result after the training indicated that children were better at this issue. This situation is also same for identification of 3D shapes category. Children could not name 3D shapes before the training, they preferred using determinant 2D shapes' names instead of actual names. They clarified that their teachers taught them so. Moreover, about recognition of 3D shapes' properties, the results indicated that children were much better and could express their understanding clearly after the training. Children could distinguish shapes, their vertices, edges, and faces. Obviously there had been a difference between before and after 3DinEC. Therefore, these results agree with current literature. For instance, Siew-Yin (2003) similarly mentioned that teachers generally preferred using prototype visual samples in instead of atypical samples in geometry education. Besides, Tsamir, Tirosh, and Levenson (2008) stated that using atypical samples would help determining the level of students' conceptual learning as well as geometrical thinking level. Therefore, participant children might have encountered with prototype samples instead of atypical ones, they could not achieve these categories before the training.

Another ability investigated is manipulation of 3D shapes' representation modes. About this issue, Duval (1998) stated that visual image of geometrical expression had a significance role in geometrical thinking. In a study, it was found out that 4 years old children could make use of their understanding about painting in geometry drawing tests (Murphy and Wood, 1981). Eryaman (2009) also constituted visualization and orientation activities about 2D representation of 3D geometric shapes, and studied the effect of these activities on 6th grade children's spatial abilities. As a result of these activities there had been improvement in children's visualization and orientation abilities. On the other hand, the present study also indicated that children had difficulties regarding this ability, especially translation of 3D shapes into isometric. Other categories were slightly better than the first category under this ability. There were difficulties children faced even after 3DinEC training. Therefore, findings of the present study indicated that although representation modes have a significant role, younger children could not make use of visual images efficiently even after a training.

Structuring 3D arrays of cubes is another ability covered. Children generally had difficulties regarding categories under this ability. Children had a limited understanding about volume. Area and volume topics are covered under calculation of the volume and the area of solids category. Heraud (1987) stated that experiences with unit squares were significant role in areas, and Battista and Clements (1996) emphasized that unit cubes could be useful for understanding the idea of volume. Children's understanding regarding these topics were very little even after the training. These results were confirmed by other researchers as well. For instance, Battista and Clements (1996) found out that the idea of volume improved as grades got higher. However, younger children in the present study were just successful in comparing the area and the volume of 3D shapes not successful in calculation of volume and area topics.

The present study focused specifically on children's 3D geometric thinking skills. The findings, on the other hand, indicated that they had a limited understanding about 3D geometric shapes as well as they had difficulties about the properties of geometric shapes (2D and 3D). Range of age of the present study was composed of children from 60 months to 72 months. This was thought to be a limitation of the study. As well, the number of children could be seen as a limitation for the study. A future study will involve more children from various ages.

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