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Analysis of Van Hiele geometric thinking levels studies in Turkey: A meta-synthesis study

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Abstract

Geometry is an important part of the mathematics curriculum. In the late 1950s, Pierre Van Hiele and Dina Van Hiele-Geldof developed a theory of learning geometry. Theory helps determine students' geometric thinking levels, design appropriate instruction for their level, and students move to the next level. This study aims to examine the studies focusing on Van Hiele geometric thinking levels (VHGTL) by using the meta-synthesis method and to reveal what kind of trend there is in this subject in mathematics education, how the studies meet the need in terms of quantity and quality. In the research, a total of 83 publications, including 33 articles and 50 theses published between 2003 and 2020, were determined to be analyzed. When the research findings were examined, it was seen that the research was conducted to determine the effects of different learning environments on the geometric thinking levels and detect the geometric thinking levels. A significant part of the studies was in the subject areas of triangles, polygons and quadrilaterals, survey and experimental studies were focused on, it was generally conducted with 7th and 8th-grade students at secondary school level, questionnaires and tests were preferred as data collection tools, the research process was based on teaching practice in only 41 publications, and the geometric thinking levels of the students were below the expected level in a significant part. Finally, the research was completed by recommending to practitioners and researchers who would work on this subject.

Keywords: Geometric thinking; Van Hiele; Meta-synthesis; Studies conducted in Turkey

1. Introduction

Most of the objects and assets that people use in their daily life while carrying out their profession or business are geometric shapes and objects. To use them effectively, it is necessary to know these items' features well and fully understand the relationships between their shape and function. In addition, basic geometric skills are required for

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simple problems (landscaping, wallpapering, model making, etc.) that people encounter daily, and geometrical thoughts feed their development. For this reason, teaching geometry is a wide strip that should be included in every grade level (Altun, 2018).

Since the beginning of the 20th century, many studies have been conducted to enrich geometry's teaching and learning processes, depending on the need to transfer geometry to future generations. The nature of geometric thinking and the results of how it can be developed have been tried to be explained (Fujita, Jones, and Yamamoto, 2004). These studies also guide educators in teaching geometry (Driscoll, DiMatteo, Nikula ve Egan, 2007; Duval, 1998; Fischbein, 1993; Herbst, 2006; Hoffer, 1981; Piaget, 1967; Van Hiele 1957). The Van Hiele theory, which directs today's geometry education and is the most cited, belongs to the Van Hiele couple in 1957. The couple, a mathematics teacher, stated that geometric thinking passes through certain levels as in mathematical operations and concepts (Teppo, 1991). These levels are sequential and hierarchical, and both the number and numbering of the levels vary. While these five levels are numbered as 0-4 in the Van Hiele couple's studies and many studies conducted after, they are numbered as 1-5 in some studies. Although there are differences regarding the order and composition of the levels, there is consensus that the levels are hierarchical and measure geometric thinking (Fuys, Geddes & Tischler, 1988; Hoffer, 1981; Mayberry, 1983; Shaughnessy & Burger, 1986; Usiskin, 1982).

Van Hiele believed that the transition from one level to the next depends not only on maturity or age but also on teaching method, geometry and educational content. He argued that it is impossible to achieve the desired learning when geometry teaching is carried out according to different level subjects than the students' level. These levels have language structure, symbols and relations (Crowley, 1987; Usiskin, 1982; Van De Walle, 2004). The levels of geometric thinking recognized by Van Hiele in his geometry teaching model at five levels are as follows:

The Visual Level: At this level, students recognize geometric shapes with their holistic view. Students recognize and name shapes based on their general visual characteristics. They can evaluate shapes according to their similarities and rank groups of shapes that look similar (Fuys et al., 1988).

The Descriptive Level: At this level, students analyze the parts of geometric shapes and the relationships between these parts. Students discover the properties/rules of a shape class experimentally (e.g., by folding, measuring, using a grid or diagram) but cannot establish a hierarchical relationship between classes (Fuys et al., 1988).

The Theoretical Level (The Informal Deduction Level): At this level, students can make connections both between the properties of shapes themselves (for example, in a quadrilateral, parallel sides require opposite angles to be equal) and classes of shapes (a square is a rectangle because it has all the properties of a rectangle) (Crowley, 1987). In addition, students can make a short definition by saying enough and necessary conditions to define a shape, instead of talking about its features for a long time. At this level, students can establish logical relationships based on informal considerations. They can follow a geometric proof but cannot prove themselves (Fuys et al., 1988).

Formal Logic: At this level, students can prove themselves in an axiomatic system. Students can use axioms, postulates, definitions and theorems in proof studies on geometry. They can determine necessary and sufficient conditions and use them in drawing conclusions and making proofs. They can also prove different theorems deductively by making use of theorems and proven axioms. For students who have reached this level, geometric shape features are a structure independent from the object and shape (Hoffer, 1981).

The Nature of Logical Laws: At this level, students identify the relationships and differences between various axiomatic systems. They comprehend Euclidean and non-Euclidean geometry and can interpret the axioms, theorems and definitions of Euclidean geometry in non-Euclidean geometry and perform applications related to these definitions (Hoffer, 1981). They can describe the effect of adding or subtracting an axiom to a certain geometric system (Vojkuvkova, 2012).

When the literature is examined, it is seen that there is a meta-synthesis study examining the studies on Van Hiele. This study was conducted by Saraçoğlu (2015), and it was a doctoral thesis in which 56 studies in total were examined. As a result of the study, it was seen that the geometric comprehension levels of primary, secondary and undergraduate students were below the expected level. Although there are similar studies on the same research topic in the literature, there are some limitations due to the lack of sufficient information about current studies and the inclusion of the studies containing the "Van Hiele Geometric Thinking Level Determination Test" only in Turkey. There is a need to examine more comprehensive and current studies on Van Hiele Geometric Thinking Levels. For this reason, it is thought that this article, which examined the data of 83 studies related to Van Hiele geometric thinking levels between 2003 and 2020, will make important contributions to the literature.

1.1. Objective and Importance of the Study

The most important step of the scientific research process for researchers is the literature review phase. At this stage, researchers need to conduct a literature review related to the subject they are studying on and determine what has been done and has not been done before. In this context, meta-synthesis studies will provide researchers with a critical and holistic perspective on their study field. When the literature is examined, it is seen that many meta-synthesis studies have been carried out in different contexts of mathematics education and at different times (Aztekin & Sener, 2015; Çiltaş, Güler & Sözbilir, 2012; Kaleli- Yılmaz, 2015; Tabuk, 2019; Türkoğlu, 2017; Ulutaş & Ubuz, 2008). In this regard, identifying trends by repeating educational research at certain time intervals will shed light on researchers who want to study in the related field (Cohen, Manion & Morrison, 2000). In addition, it is emphasized that new studies to be carried out are shaped as a result of previous studies and following current studies is of importance (Varışoğlu, Şahin & Göktas, 2013). For this purpose, in this study, it was aimed to examine the studies conducted in Turkey between 2003 and 2020 on the levels of Van Hiele geometric thinking using the meta-synthesis method, and what kind of a trend there was in this subject in mathematics education and how the studies responded to the need in terms of quantity/quality, what kind of new studies were needed to be done. In this context, it was desired to evaluate the general situation on the subject and present the existing information systematically within the framework of certain themes. For this purpose, answers were sought for the following questions.

1- What are the purposes of VHGTL research?

2- In which subject areas has VHGTL been studied?

3- What are the methods used to achieve these purposes in studies on VHGTL?

4- What are the sample features preferred in research on VHGTL?

5- What are the data collection tools used in VHGTL research?

6- What are the similarities and differences between the teaching practices used in VHGTL research?

7- What are the results of research on VHGTL?

8- What are the recommendations and, if any, deficiencies identified in the research on VHGTL?

1.2. Limitations of the Study

This research covers the studies conducted in Turkey by Turkish researchers between 2003 and 2020 and is limited to 83 studies given in the bibliography. Data not directly related to Van Hiele geometric thinking levels were not included in the study. In addition, if the study with the same name was published as both a thesis and an article, only the studies published as an article were discussed to avoid data repetition. In addition, the assertions on the subject were not included in the research.

2. Method

2.1. Research Model

The Meta-synthesis method was used in this study, which aimed to evaluate the studies on Van Hiele geometric thinking levels in mathematics education. Meta-synthesis study is based on examining, synthesizing and interpreting researches focusing on the same subject in a particular field according to the determined themes or templates. These studies provide an important resource for teachers, practitioners and researchers in terms of addressing a certain subject according to similarities and differences, synthesizing its different dimensions qualitatively, and involving further studies (Çalık, Ayas &Ebenezer, 2005; Gül & Sözbilir, 2015). In this research, the meta-synthesis method was used since it was aimed to analyze the geometric thinking studies carried out in Turkey by qualitative methods. It is presented in this section how the 83 studies reviewed were selected and how the data were analyzed.

2.2. Data Collection, Criteria of Inclusion in the Study and Analysis

The analyzed studies were obtained from the National Thesis Center of the Council of Higher Education (CoHE), TÜBİTAK ULAKBİM DergiPark and Google Academic search engine databases. The search terms (key concepts) used for literature searches are "Van Hiele", "geometric thinking," and "geometric thinking levels". Studies on Van Hiele geometric thinking levels specified in the title and summary of the research were examined within the scope of this research. The studies within the scope of the research were determined according to the criterion sampling method, which is one of the purposive sampling methods. The criteria in the study are;

- ✓ Being conducted in Turkey,
- ✓ Being conducted by Turkish researchers,
- ✓ The language of writing is Turkish,
- ✓ Including the keywords specified in the thesis and articles.

In addition, if any study was published as a thesis and an article simultaneously, only the studies published as an article were included in the research so that the data would not be repeated. In addition, assertions of the congress and other academic events like congress weren't included in this research. 50 graduate theses and 33 articles on Van Hiele geometric thinking levels, which were determined to be carried out in Turkey between 2003 and 2020 due to the review, were analyzed.

Each study examined within the scope of the research was first read in detail, coded according to each theme in line with the research problems, and recorded in the computer environment. Each study examined was coded as A1, A2, A3,…, A83. The data were reviewed, and unnecessary parts were removed.

2.3. Validity and Reliability of the Research

After the studies to be included in the research were determined, the coding process was started. To avoid any errors in the coding process, the time-dependent coding reliability method and inter-researchers coding compatibility methods were used together. First, separate files were created in the computer environment for each sub-problem by the first researcher. All the data related to the problems were recorded in these files, and coding was done over a long period. Nearly a month after the coding was done, the coding was done again, and it was observed that there was a significant consistency between the coding during this one month. Afterward, the coding was checked by the second researcher, deficiencies and necessary changes were determined. After these determinations, the two researchers came together, and the final decision was made on the coding. In this way, the coding for each sub-problem continued until there was complete consistency between the two researchers.

3. Findings

In this section, the findings obtained as a result of the data analysis are given in line with the research problems.

3.1. Purposes of the Studies Examined

Table and explanations regarding the purposes of the studies examined within the scope of the research are given below.

Table 1. Data on the Purposes of the Examined Studies

Purposes	**Studies**	**Frequency**
Determining the effects of different learning environments on geometric thinking levels	A3, A10, A14, A17, A22, A23, A24, A26, A28, A35, A39, A41, A45, A46, A51, A52, A53, A58, A62, A63, A64, A71, A72, A73, A74, A75, A77, A78, A80	29
Determining geometric thinking levels	A5, A6, A9, A11, A12, A15, A16, A19, A20, A21, A25, A29, A33, A34, A36, A37, A40, A47,A48, A53, A55,A59, A61, A66, A68, A83	26
Examining the relationship between geometric thinking levels and geometry achievements	A4, A13, A16, A25, A33, A37, A48, A50, A79	9
Examining geometric thinking levels in terms of different variables (geometry self-efficacy beliefs, demographic variables)	A5, A6, A12, A20, A34, A36, A38, A59	8
Examining the relationship between geometric thinking levels and attitudes towards geometry	A5, A9, A11, A30, A76	5
Determining the effect of teaching based on the Van Hiele model on success	A22, A31, A44, A49, A71	5
Examining the relationship between geometric thinking levels and self-efficacy beliefs towards geometry	A5, A12, A19	3
Examining the relationship between geometric thinking levels and critical or spatial skills	A32, A66, A76	3
Determining the relationship between geometric reasoning and Van Hiele geometric thinking levels	A1, A69	2
Examining problem-solving strategies in terms of Van Hiele thinking levels	A7, A8	2
Examining the relationship between geometric thinking levels and intelligence areas	A15, A57	2
Examining the effects of geometric drawing practices on geometric thinking levels	A40, A43	2
Examining the effect of using concept maps on geometric thinking levels	A2	1
Examining the relationship between Van Hiele geometry comprehension levels and proof-writing achievements	A18	1
Determining the relationship between belief in origami and geometric thinking level	A21	1
Examining the views and practices of teachers on teaching geometry according to Van Hiele levels	A27	1
Constructing geometric understanding levels for spherical geometry and determining their relationship with Van Hiele levels	A42	1
Examining the relationship between classification skills of polygons and geometric thinking levels	A47	1
Associating the syntax and semantic components of the mathematical language with Van Hiele geometric thinking levels	A54	1
Examination of geometric thinking levels and brain dominance in terms of some variables	A56	1
Examining the relationship between algebraic and geometric thinking levels	A60	1
Examining the relationship between Van Hiele geometric thinking levels and visual proof skills	A65	1
Revealing the results of the researches conducted in the field of geometric thinking in Turkey	A67	1
Determining the effect of geometric thinking levels on the success of constructing and drawing geometric structures	A70	1
Examining the effect of the prepared professional development program on the geometric thinking level	A81	1
Determining errors and misconceptions according to geometric thinking levels	A82	1

An important part of the studies examined within the research scope was conducted to determine the effects of different learning environments on geometric thinking levels and determining geometric thinking levels. Besides, in general, there were also studies examining the relationship between geometric thinking levels and geometry achievements and examining geometric thinking levels in terms of different variables. It was seen that studies conducted for other purposes were less in number.

3.2. Subject Areas in the Studies Examined

Figure and explanations regarding the subject areas of the studies examined within the scope of the research are given below.

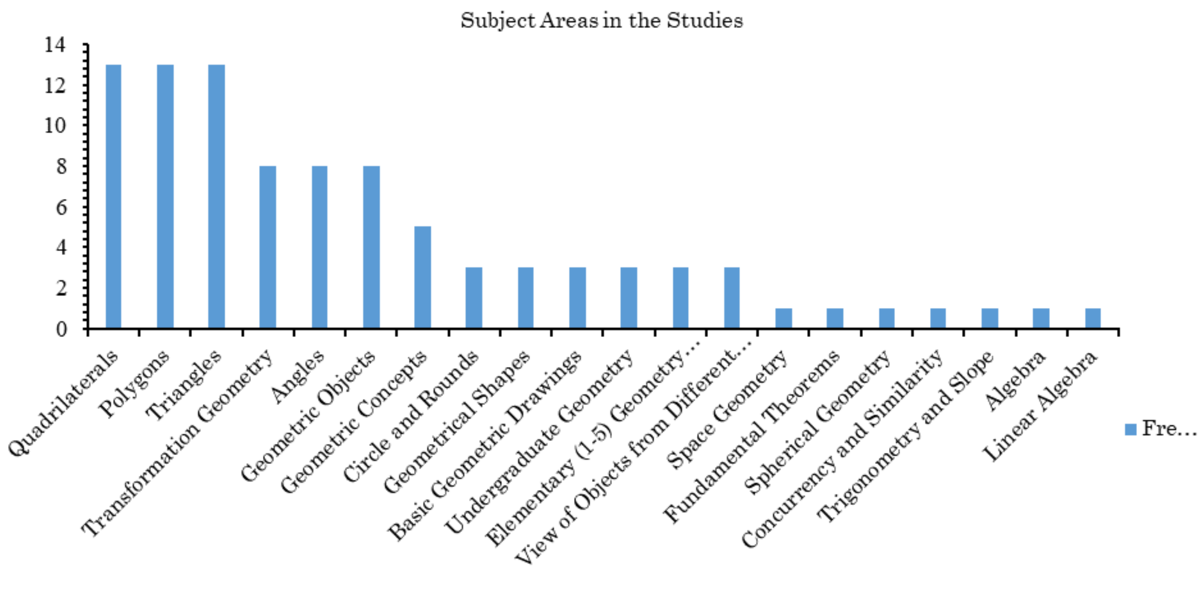

Figure 1. Subject Areas in the Studies

When Figure-1 was examined, it was seen that the studies on Van Hiele were mainly on Triangles, Polygons and Quadrilaterals, and there were fewer studies on Linear Algebra, Algebra, Trigonometry and Slope, Concurrency and Similarity, Space Geometry, Spherical Geometry and Fundamental Theorems. A15, A34, A60, A66, A83 coded studies were not included in any subject area since they were conducted by scanning method.

3.3. Methods Used in the Examined Studies

Table and explanations regarding the purposes of the studies examined within the scope of the research are given below.

Table 2. Data on Methods of the Examined Studies

Research Method		**Studies**	**Frequency**
Quantitative	Experimental Method	A3, A10, A17, A21, A22, A24, A28, A31, A35, A39, A40, A44, A45, A46, A49, A51, A52, A53, A58, A62, A63, A64, A70, A71, A72, A73, A74, A77, A78, A80, A81	31
	Survey	A2, A4, A5, A6, A9, A11, A12, A15, A16, A19, A25, A29, A30, A33, A34, A36, A37, A38, A47, A48, A55, A56, A57, A59, A60, A66, A68, A69, A70, A79, A82	31
	Correlational research model	A65, A76	2
	Quantitative research	A13, A20	2
Qualitative	Case Study	A1, A7, A8, A18, A32, A38, A75	7
	Qualitative research	A50, A54	2
	Phenomenography	A27	1
	Teaching Experiment	A41	1
	Meta-synthesis	A67	1
Action research		A23, A26, A42, A43	4
Mixed Method		A14, A61, A83	3
Total			85

When Table-2 was examined, it was noteworthy that in a significant part of the studies conducted on Van Hiele, experimental and screening methods, which are among the quantitative research methods, were preferred. In studies in which experimental method was used, the effect of different learning environments on geometric thinking levels was generally tried to be determined. In contrast, in studies in which the scanning method was used, the relationship between geometric thinking levels and geometry achievements was examined. As can be seen, few studies were conducted with phenomenography, teaching experimentation and meta-synthesis methods. In addition to these, it was seen that general names were given to the methods as quantitative research (A13, A20) and qualitative research (A50, A54) in some studies, and methods were not specifically specified. In addition, Survey and Case Study methods were used in the A38 coded study and the Experimental and Survey methods in the A70 coded study together. There are also studies coded A14, A61 and A83 in which mixed method is preferred.

3.4. Sample Group of the Examined Studies

Table and explanations regarding the sample group of the studies examined within the scope of the research are given below.

Table 3. Data on Samples of the Examined Studies

Sample Type	Sample level	Study	Frequency
Primary School	4th Grade	A74	1
Secondary School	5th Grade	A23, A34, A41, A49, A50, A53, A70	7
	6th Grade	A22, A55, A80	3
	7th Grade	A3, A17, A25, A26, A33, A45, A46, A47, A58, A62, A75, A82	12
	8th Grade	A5, A15, A16, A24, A32, A37, A48, A54, A60, A64, A71, A76, A79	13
	Mixed	A12, A30, A35, A38, A43, A63, A83	7
	Hearing-impaired	A19, A77, A78	3
	Highly gifted	A66	1
High School	9th Grade	A7, A8, A39, A44	4
	10th Grade	A52	1
	11th grade	A4	1
	Mixed	A18	1
	Not specified	A51	1
Teacher Candidate	Grade	A9, A10, A29, A31, A36, A61, A68, A72	8
	Mixed	A2, A11, A13, A42, A56, A57, A59	7
	Primary School Mathematics	A14, A20, A21, A28, A65, A69, A73	7
	Mathematics Teaching	A40	1
Teacher	Grade	A6, A27, A68	3
	Primary School Mathematics	A81, A83	2
	High School Mathematics	A1	1
Other	Literature	A67	1

It was seen that the sample groups of the studies examined within the scope of the research were predominantly at the secondary school level, they were conducted with eighth and seventh-grade students, and the studies conducted with primary and high school teachers were few.

3.5. Data Collection Tools Used in the Examined Studies

Table and explanations regarding the data collection tools of the studies examined within the scope of the research are given below.

Table 4. Data on Data Collection Tools of the Examined Studies

Data Collection Tool Type	Data Collection Tool	Study	Frequency
TEST	Van Hiele Geometric Thinking Test	A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14, A15, A16, A17, A18, A19, A20, A21, A22, A23, A24, A25, A28, A29, A30, A31, A32, A33, A34, A35, A36, A37, A38, A39, A40, A42, A44, A45, A46, A47, A48, A49, A51, A52, A53, A56, A57, A58, A59, A60, A61, A62, A63, A64, A65, A66, A68, A69, A70, A72, A73, A74, A76, A77, A78, A79, A80, A81, A82, A83	73
	Geometry Achievement Test	A10, A14, A16, A17, A22, A24, A25, A31, A33, A35, A37, A43, A44, A48, A49, A51, A64, A70, A71, A74, A77, A78, A79, A80	24
	Spatial Ability Test	A1, A45, A65, A66, A70, A76, A79	7
	Geometry Proof Test	A18, A51, A65	3
	Van Hiele Transformation Geometry Thinking Levels Test	A26	1
	Geometry Readiness Test	A29	1
	Geometric Object Test	A32	1
	Mental Rotation Test	A32	1
	DeterminationTest of Shape Construction Skill Levels	A38	1
	Spherical Geometry Understanding Levels Exam	A42	1
	Quadrilateral Classification and Identification Exam	A58	1
	Hierarchical Chart Exam	A58	1
	Algebraic Thinking Test	A60	1
	Geometric Reasoning Problems Test	A69	1
	Error and Misconception Identification Test	A82	1
SCALE/ SURVEY	Geometry Attitude Scale	A5, A9, A11, A22, A29, A30, A49, A74, A76, A77	10
	Self-Efficacy Scale for Geometry	A5, A12, A17, A19, A29	5
	Mathematics Attitude Scale	A17, A40, A79	3
	Critical Thinking Skills Assessment Tool for Angles and Polygons	A17	1
	Origami Belief Scale	A21	1
	Attitude Survey Towards Geometric Drawings	A43	1
	Polygon Identification and Classification Scale	A47	1
	Cornell Critical Thinking Skills Scale	A66	1
INTERVIEW/ DISCUSSION	Interview (Discussion)/ Interview Form	A1, A7, A8, A14, A17, A23, A27, A32, A35, A38, A41, A42, A43, A45, A46, A54, A55, A58, A69, A75, A83	21
OBSERVATION OBSERVATION FORM	Observation	A26, A27, A42, A74, A75	5
DOCUMENT	Worksheets/Activity	A3, A14, A17, A32, A41, A50, A58, A62, A63	9
	Screen recordings/ Video	A14, A55	2
	Diary	A26	1
	Field Notes	A58	1
	Literature	A67	1
INVENTORY	Multiple Intelligence Inventory	A15, A57	2
	Brain Dominance Inventory	A56	1
OTHER	Open-ended questions	A1, A7, A8, A23, A39, A55, A74, A75	8
	Personal Information Form	A6, A12, A19, A20, A36, A57	6
	Rubric/ Evaluation Form	A2, A17	2
	Activity Paper	A2	1
	Product Selection File	A10	1
	Module	A17	1

It was seen that the test was used in a significant part of the studies examined within the scope of the research. The biggest share among the tests belonged to the "Van Hiele Geometric Thinking Test". At the same time, the studies that were given under the name of exam in the studies examined were also included under this title. Since scales and surveys were often used interchangeably in studies, giving them under the same title was appropriate. Most of the studies in which scales/surveys were used aimed to examine the relationship between the level of geometric thinking and a variable (A5, A9, A21, A43, A47).

As a result of the examinations, it was determined that the interview/discussion form was used in 21 studies. The most used were (A1, A23, A38, A41, A42, A43, A46, A54, A58) clinical interview and (A7, A8, A14, A32, A35, A55, A69, A83) semi-structured interview technique. The individual interview technique was used as a data collection tool in A27 due to the nature of phenomenographic work. It was observed that studies that preferred observation/observation form as a data collection tool (A27, A42, A75) were used together with the interview technique. When Table-4 was examined, it was seen that there were 9 studies using worksheets/activities. While A3 and A17 were based on the problem, A14 and A58 on DGS, A41 on Van Hiele model, A62 on Teaching by Discovery and they used worksheets as data collection tool, A32 and A50 used interviews to determine students' geometric thinking levels and knowledge of the subject. In the examinations, few studies were found which used data collection tools of screen recordings, diary, field notes and literature.

3.6. Teaching Practices Used in the Examined Studies

Table and explanations regarding the teaching practices used within the scope of the research are given below.

Table 5.Teaching Practices Used in the Examined Studies

Teaching Practice	**Studies**	**Frequency**
Teaching with DGS	A14, A24, A35, A46, A51, A58, A64, A74, A78	9
Teaching Based on the Van Hiele Model	A22, A31, A44, A49, A71	5
Computer-Aided Instruction	A28, A41, A45, A63, A73	5
Teaching Supported by Concrete Materials	A28, A46, A58, A74	4
Teaching with Constructivist Learning	A10, A22, A31	3
Teaching with Origami Activities	A21, A23, A39	3
Teaching with the 5E Learning Model	A26, A52, A80	3
Teaching with a PBL Approach	A3, A17	2
Teaching with Geometric Drawing Methods	A40, A43	2
Teaching with Concept Maps	A2	1
Teaching by Discovery	A62	1
Concrete and Virtual Manipulative Supported Education	A70	1
Teaching with RBC Theory	A75	1
Teaching with Professional Development Model	A81	1

DGS: Dynamic Geometry Software PBL: Problem-Based Learning RBC: Recognizing-Building with-Constructing Abstraction Theory

Among the studies examined, the research process was based on teaching practice in only 41 studies. Among these, while the most frequently used were "Teaching with DGS", "Teaching Based on the Van Hiele Model" and "Computer-Aided Instruction"; "Teaching with Concept Maps", "Teaching by Discovery", "Concrete and Virtual Manipulative Supported Education", "Teaching with RBC Theory" and "Teaching with Professional Development Model" were less preferred.

While an increase was observed in the geometric thinking levels of the students in most of the teaching practices with DGS (A14, A46, A51, A74, A78), it was not effective in a significant part of them (A24, A35, A58, A64). It was seen that students were effective in improving their geometric achievement levels in A31 and A71 coded studies in the weekly lessons of the teaching constructed with an experimental design according to Van Hiele geometric thining levels. At the same time, it was observed that the teaching according to the Van Hiele model was more effective than the traditional method (A22, A31, A44, A49, A71). In most of the studies based on Computer-Aided Instruction, an increase was observed in students' geometric thinking levels (A28, A41, A63). However, in the A73 coded study in which 9-week linear algebra was taught, it was concluded that although the courses were actively taught with mathematics software, the teaching was ineffective. It has been observed that teaching supported by concrete materials (geometric board and strips, dotted paper) effectively increased students' geometric thinking levels (A28, A46, A74). As a result of the applications that lasted for two weeks, the teaching in the A58 coded study, which was supported by concrete materials, was insufficient. The Constructivist Learning design consisted of activities that lasted six weeks for the geometry subject of the Basic Mathematics II course, and the diaries written by the students after each lesson were effective in Van Hiele's geometric thinking levels (A10). As a result of the 3-week (12 class hours) education on angles and triangles, it was observed that it was effective in improving students' attitudes and achievements towards geometry (A22). Geometric thinking and readiness levels improved during the 6-week education period on various geometry subjects (A31). Teaching with origami activities effectively increased students' geometric achievement levels (A21, A23, A39). Teaching with appropriate teaching activities prepared according to the 5E learning cycle model at different grade levels and following the action plans that would last for weeks had a positive effect on the geometric thinking levels of the students (A26, A52, A80). It was determined that the geometric thinking levels of the students increased in the learning groups in which the problem-based learning approach was adopted, and the various designed activities were taught (A3, A17).

In the A40 and A42 coded studies using compass-ruler, it was observed that the Van Hiele geometry comprehension levels were higher than the control groups. As a result of the applications made in the A2 coded study, in which geometry concept information and concept relations were examined with the help of concept maps, it was seen that geometric thinking levels were reflected in concept maps; therefore, concept maps played an active role in revealing geometry information. As a result of the fact that the A62 concrete and virtual manipulative supported teaching practices made through invention were a part of the teaching processes, it was determined that it positively affected geometric thinking levels. The students' knowledge formation processes were examined in the case study conducted with two students who had different geometric thinking levels. It was seen that these students' mathematical thinking and knowledge formation processes were different (A75). It was determined that the professional development program prepared to help secondary school students and teachers acquire geometric thinking habits was not effective in increasing the students' Van Hiele geometric thinking levels (A81). Such different teaching practices are important in revealing new approaches that can provide solutions to the problems that occur in the geometry teaching process.

3.7. Results from the Examined Studies

The results obtained from the studies examined were examined in detail, and the results directly related to the Van Hiele theory are given in the table below.

Table 6. Results from the Examined Studies

Results	Studies	Frequency
The geometric thinking levels of the students were below the expected level.	A4, A5, A6, A12, A15, A16, A18, A19, A20, A25, A29, A30, A32, A33, A34, A36, A37, A38, A44, A48, A54, A56, A57, A59, A60, A61, A65, A67, A72, A81, A83	31
Different learning environments were effective in increasing students' geometric thinking levels.	A3, A10, A14, A17, A21, A23, A26, A28, A31, A35, A39, A41, A46, A51, A52, A62, A63, A70, A71, A74, A77, A78, A80	22
Some variables **did not affect** the geometric thinking level (gender, age, branch, high school type).	A5, A6, A11, A12, A20, A25, A37, A57, A59, A67, A68, A76	12
The geometric thinking level **was affected** by some variables (gender, parental education level, preschool education).	A5, A12, A30, A34, A36, A48, A68, A76, A79	9
Different learning environments were not effective in increasing students' geometric thinking levels.	A24, A35, A53, A58, A64, A73	6
There was a significant relationship between geometric thinking levels and spatial ability.	A32, A45, A66, A76, A79	5
There was a significant relationship between geometric thinking levels and attitudes towards geometry.	A9, A11, A76, A79	4
There was a significant relationship between the achievement test developed by the researchers and the Van Hiele geometry test.	A16, A25, A37, A48	4
The current geometry learning program was insufficient to provide students with high-level thinking skills.	A52, A72, A83	3
There was a significant relationship between geometric reasoning and Van Hiele geometric thinking levels.	A1, A69	2
Geometry lessons did not significantly affect students' geometric thinking levels.	A4, A13	2

Problem-solving strategies differed according to Van Hiele thinking levels.	A7, A8	2
There was a low-level positive and significant relationship between students' geometric thinking levels and their self-efficacy beliefs towards geometry.	A12, A19	2
With the increase in grade levels, students' geometric thinking levels increase.	A12, A30	2
There was a significant relationship between students' geometric thinking levels and intelligence types (logical, visual and verbal intelligence).	A15, A57	2
Geometric drawing applications increased the level of geometric thinking.	A40, A43	2
Students' geometric thinking levels were related to their geometry achievement.	A50, A55	2
Different learning environments were not effective in increasing the geometric thinking level of the students with hearing impairment.	A77, A78	2
The use of concept maps in geometry teaching increased Van Hiele geometric thinking level.	A2	1
There was a moderate positive correlation between Van Hiele levels and proof-writing skills.	A18	1
Education given according to Van Hiele geometric thinking levels positively affected students' attitudes.	A22	1
Experience played an important role in teaching based on Van Hiele levels.	A27	1
The reliability of the van Hiele geometric thinking test was low.	A36	1
As the level of geometric thinking increased, the level of shape construction skills also increased.	A38	1
There was a moderate relationship between spherical geometry understanding levels and Van Hiele levels.	A42	1
Teaching with the Van Hiele model was more permanent than the traditional method.	A44	1
There was a significant relationship between polygon classification skills and geometric thinking levels.	A47	1
The use of mathematical language affected geometric thinking levels.	A54	1
Teaching according to geometric thinking levels was effective in establishing geometric relationships among students.	A55	1
There was no relationship between brain dominance and Van Hiele geometric thinking levels.	A56	1
There was a significant relationship between geometric and algebraic thinking levels.	A60	1
There was a significant relationship between visual proof skill and geometric thinking level.	A65	1
It was seen that teaching experience was no effective variable on teachers' Van Hiele geometric thinking levels.	A68	1

Mathematical thinking and knowledge formation processes of students at different geometric thinking levels were also different.	A75	1
The professional development model was not effective in increasing the geometric thinking levels of the students.	A81	1
The students with low Van Hiele geometric levels had more errors and misconceptions.	A82	1

When the results obtained from the studies were examined, it was seen that the students' Van Hiele geometric thinking levels were below the expected level in a significant part (A4, A5, A6, A12, A15, A16, A18, A19, A20, A25, A29, A30, A32, A33, A34, A36, A37, A38, A44, A48, A54, A56, A57, A59, A60, A61, A65, A67, A72, A81, A83). In most of the studies in which the learning process was based on different teaching practices (A3, A10, A14, A17, A21, A23, A26, A28, A31, A35, A39, A46, A51, A52, A62, A63, A70, A71, A74, A77, A78, A80) it was observed that geometric thinking levels increased while it did not affect geometric thinking levels in some studies (A24, A35, A53, A58, A64, A73). In addition, studies on hearing-impaired students found that different teaching practices did not increase students' geometric thinking levels (A77, A78).

Considering the studies, it was found that the level of geometric thinking level was not affected by some (gender, age, branch, high school type) variables (A5, A6, A11, A12, A20, A25, A37, A57, A59, A67, A68, A76). In contrast, it was affected by variables such as gender, parental education level, and preschool education in some studies (A5, A12, A30, A34, A36, A48, A68, A76, A79). In the studies examined, it was seen that there was a significant relationship with geometric thinking level and spatial ability (A32, A45, A66, A76, A79), attitude towards geometry (A9, A11, A76, A79), self-efficacy beliefs towards geometry (A12, A19), geometric reasoning (A1, A69), polygon classification skill (A47), algebraic thinking levels (A60), visual proof skills (A65), intelligence types (A15, A57) and proof-writing skills (A18). In addition, there was no relationship between brain dominance and Van Hiele geometric thinking levels (A56).

In the re-examined studies, it was determined that there was no significant relationship between the geometric thinking level and geometry lessons (A4, A13). In contrast, in some studies, it was found that the geometric thinking level was associated with geometry achievement (A50, A55). It was concluded that the existing geometry learning program (A52, A72, A83) and the prepared professional development program (A81) did not increase students' geometric thinking levels. It was observed that the geometric thinking levels of the students increased with their grade levels (A12, A30), the use of concept maps (A2) and geometric drawing applications (A40, A43) in geometry teaching. Moreover, it was concluded that teaching based on the Van Hiele model was more permanent than teaching with traditional methods (A44), positively affected students' attitudes (A22), was effective in establishing geometric relationships (A55), and experience (A27) played an important role in the teaching process. At the same time, it was observed that the use of mathematical language affected the geometric thinking levels (A54). However, it was determined that teaching experience was not an effective variable on teachers' geometric thinking levels (A68).

Studies showed a significant relationship between the achievement test developed by the researchers and the Van Hiele geometry test (A16, A25, A37, A48). Also, it was concluded that the reliability of the Van Hiele geometric thinking test was low in a study (A36). In the other studies examined, it was concluded that problem-solving strategies differed according to Van Hiele thinking levels (A7, A8). Students' mathematical thinking and knowledge formation processes at different geometric thinking levels were different (A75). There was a moderate relationship between their spherical geometry understanding and geometric thinking levels (A42). In addition, it was observed that as the level of geometric thinking increased, the level of shape construction skills increased (A38), while students with low geometric thinking levels had more errors and misconceptions (A82).

3.8. Recommendations Obtained from the Studies Examined

Recommendations obtained from the studies were examined in detail, and those directly related to the Van Hiele Theory are given in the table below.

Table 7. Findings and Recommendations Obtained from the Studies Examined

Recommendations	**Studies**	**Frequency**
In-service training and seminars can be given to teachers about Van Hiele geometric thinking levels.	A5, A6, A12, A16, A22, A25, A33, A48, A49, A53, A54, A57, A67, A68, A77, A81	16
Geometry teaching can be done to increase students' geometric thinking levels after determining their current levels.	A4, A5, A12, A16, A47, A67	6
The mathematics curriculum should be revised according to Van Hiele thinking levels.	A6, A9, A26, A67, A68	5
The Van Hiele test can be revised for language and intelligibility.	A30, A36, A67	3
A scale equipped with mathematical reasoning and spatial problems can be developed to determine the relationship between spatial thinking skills and geometric thinking skills.	A67, A73	2
Courses can be included in undergraduate education programs to increase Van Hiele geometric thinking levels.	A46	1
Another scale can be developed to measure the geometric thinking skills of preschool children.	A67	1
Measurement tools different from the Van Hiele test can be developed to measure and evaluate geometric reasoning skills.	A69	1

The recommendations obtained from the studies examined are presented in detail in Table-7. The suggestions given in the table are the situations that are directly related to the subject and determined for structuring. When the recommendations were examined, it was seen that a significant part of them recommended that in-service training and seminars could be given to teachers on Van Hiele geometric thinking levels (A5, A6, A12, A16, A22, A25, A33, A48, A49, A53, A54, A57, A67, A68, A77, A81). Also, they recommended that the current geometric thinking levels of the students could be determined. Geometry teaching could be done to increase the level (A4, A5, A12, A16, A47, A67), and the mathematics currency should be revised according to their Van Hiele thinking levels (A6, A9, A26, A67, A68). In addition, it was stated that the Van Hiele test should be revised in terms of language and intelligibility (A30, A36, A67). In addition, it was stated that other measurement tools could be developed to measure the geometric thinking skills (A67) and spatial thinking skills of preschool children (A67, A73).

4. Discussion, Conclusion and Recommendations

In this section, the results obtained from the research are discussed in line with the research problems and recommendations are presented. When the findings were examined, it was seen that the studies were generally conducted to determine the effect of different learning environments on geometric thinking levels, to determine geometric thinking levels, and to examine the relationship between geometric thinking levels and geometry achievements. It is very important to determine the geometric thinking levels of the students because teaching a lesson above or below the geometric thinking level of the student is the biggest obstacle to learning. In addition, there are many studies examining the relationships between geometric thinking levels and attitudes, self-efficacy beliefs, critical thinking, geometric reasoning and algebraic thinking skills.

It was seen that a significant part of the studies on Van Hiele was in the subjects of triangles, polygons and quadrilaterals. The reason for this situation can be shown as the fact that the subjects in the Van Hiele test mainly include them. In addition, it was determined that subjects such as linear algebra, algebra, trigonometry and slope, parity and similarity, space geometry, spherical geometry and fundamental theorems were studied in a limited number.

When the studies were examined, it was seen that the survey studies included in the quantitative researches and experimental studies were mainly focused on. Ross, Morrison, and Lowther (2010), Küçük, Aydemir, Yildirim, Arpacik, and Göktaş (2013) analyzed the methods used in researches and determined that quantitative research was used in a significant part of the studies and this result is in parallel with this study. In addition, in the studies examined by Parlakkılıç and Güldüren (2019) and Solmaz and Gökçearslan (2016), it is seen that the experimental design was mostly preferred, similar to this study. In studies where the experimental method was preferred, it was aimed to compare different learning environments (such as teaching with DGS, teaching based on Van Hiele model, computer-aided teaching) with traditional teaching and to determine the effect of this on geometric thinking levels. It was seen that the teachings were effective in most of these studies. It can be said that geometry lessons carried out with applications that allow students to learn actively effectively increase their geometric thinking levels. It can be found that the geometric thinking levels of the students develop because they gain high-level achievements such as making sense and establishing relationships, developing different perspectives on the subject, and reasoning skills instead of memorizing information. It is seen that the Van Hiele Geometric Thinking Test was used in all of the survey studies. In this study, it was seen that quantitative methods were frequently preferred, but as Küçük et al. (2013) emphasized in their studies, quantitative methods have begun to lose their power. In fact, it has been noticed that the tendency towards qualitative methods has increased in studies conducted abroad in recent years (Kelly & Lesh, 2000; Masood, 1997). Therefore, since qualitative research methods provide the opportunity to examine the causes of the problems in more depth, it is thought that the frequent preference of these research methods will enrich the researches in the field of mathematics education in Turkey. In addition, it was seen that there were very few studies using phenomenography, teaching experiments and meta-synthesis methods. It is foreseen that more studies using these methods will contribute to the field.

Looking at the samples of the studies examined, it was seen that they studied at all levels, including primary school, secondary school, high school, teacher candidate and teacher. It was determined that 46 of 83 studies were conducted with 7th and 8th-grade students at the secondary school level. This finding is in parallel with the research results conducted by Lubiensky and Bowen (2000) and Ulutaş and Ubuz (2008). This may be because the Van Hiele test questions are compatible with the 7th and 8th grades according to the secondary school mathematics course contents. In the studies examined, it was seen that the number of studies conducted with teacher candidates was quite high. The reasons for this are that the teacher candidates are easily accessible by the instructors and that all the questions in the Van Hiele Geometric Thinking Test can be applied to the teacher candidates. In addition, in the studies conducted, it was seen that teacher candidates and teachers had lower geometric thinking levels than they should have (Asik-Unal & Vezne, 2021; Bal, 2012; Durmuş, Toluk & Olkun, 2002). The low geometric thinking level raises the suspicion that there are some problems in geometry field knowledge. Considering that teacher candidates should carry out effective and meaningful teaching on geometry subjects when they start their profession, measures should be taken urgently to increase these thinking levels in the current situation of teachers. At this point, it is thought that pre-service and in-service training activities may increase geometric thinking levels.

Since most of the studies examined were quantitative, the most used data collection tools were surveys and tests. The Van Hiele Geometric Thinking Level Test was used in 73 of the 83 studies (excluding the studies with the codes A27, A41, A43, A50, A54, A55, A67, A71, A75). This test was developed by Usiskin (1982), and its Turkish adaptation and validity-reliability studies were performed by Baki (1994; 2006) and Duatepe (2000). The test translated into Turkish by Duatepe (2000) was used in a significant part of the studies (A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, A11, A13, A14, A15, A16, A17, A19, A20, A21, A22, A23, A24, A25, A28, A29, A30, A31, A32, A33, A34, A35, A36, A37, A38, A40, A48, A49, A53, A56, A57, A59, A60, A61, A63, A64, A65, A66, A68, A69, A70, A72, A73, A76, A77, A78, A79, A80, A81, A83). There are many studies adapted into Turkish by Baki (1994, 2006) (A18, A42, A44, A45, A46, A58, A74, A82). The Van Hiele Geometric Thinking Level Test developed by Fidan (2009) was used in the A62 coded study and by Özcan (2012) in the A12 and A47 coded studies. In addition, although the Van Hiele test was used in studies coded A39, A51 and A52, it was used without reference. At the same time, although Duatepe (2001) was referenced in the text in the studies coded A22, A24, A31, A37, A64 and A72, it was seen that the bibliographies of the studies were dated 2000. In this context, it is important to develop alternative tests for geometric thinking levels and discuss the current test's deficiencies. It is also thought that conducting curriculum studies for different grade levels related to Van Hiele geometric thinking levels would be beneficial.

Among the studies examined, only 41 studies were based the research process on teaching practice. The most preferred among these was the design of teaching with dynamic geometry software in lessons. Studies of this kind take place over a long period and provide useful data to reveal the change in this process. Increasing such studies is important to increase the geometric thinking levels of students and teacher candidates. Additionally, the different teaching practices used are important in solving some problems that arise in the geometry teaching process.

When the results obtained from the studies focusing on Van Hiele were examined, it was seen that the geometric thinking levels of the students were below the expected level in a significant part. Unfortunately, this situation causes students not to understand mathematics and geometry lessons and naturally fail. In addition, it is thought that the lower than expected geometric thinking levels are effective in the low achievement level of the students in Turkeyin exams such as TIMSS and PISA. Already, it is clear that we fail mostly in geometry sub-dimension in TIMMS and space and shape dimension after the numerical sub-dimension in PISA. This situation has led to changes in the curriculum of Turkey. In the renewed program, geometry is included in every grade level (MoE, 2018). In addition, it was observed that different learning environments were effective in increasing students' geometric thinking levels (Kaleli- Yılmaz & Koparan 2016). However, there are also studies showing that different learning environments were not effective in increasing students' geometric thinking levels (A53, A73). When we looked at these studies, it did not have a positive effect on the geometric thinking levels of the students in the study coded A53 that a teaching plan that provided the transition between Van Hiele geometric thinking levels was not implemented, in the study coded A73, the items of the Van Hiele geometric thinking levels scale included synthetic geometry topics and the instruction given was not related to the questions.

When the geometric thinking levels were examined in the context of various variables, it was concluded that it was not affected by variables such as gender, age, branch, and high school type. In contrast, it was affected by the educational status of the parents, and preschool education. It is predicted that the effect of preschool education on the cognitive development of children will also have a positive effect on their geometric thinking levels at later ages. It was seen that the educational status of the parents played an active role in the educational status of the students (Dam, 2008; Ötken & Anıl, 2016). Considering this situation, the effect on geometric thinking levels is not surprising because parents are both guides and good role models in this process.

It is seen that many recommendations were given in the studies carried out. The most repetitive of these recommendations was that in-service training and seminars could be given to teachers about Van Hiele geometric thinking levels. In addition to these, the shortcomings and revisions that need to be made, both suggested in the literature and determined as a result of this study, can be as follows: Geometry teaching can increase students' geometric thinking levels by determining their current level. The mathematics curriculum should be revised according to Van Hiele thinking levels. The Van Hiele test can be revised for language and intelligibility. A scale equipped with mathematical reasoning and spatial problems can be developed to determine the relationship between spatial thinking skills and geometric thinking skills. Another scale can be developed to measure the geometric thinking skills of preschool children. Measurement tools different from the Van Hiele test can be developed to measure and evaluate geometric reasoning skills. Courses can be included in undergraduate education programs to increase Van Hiele geometric thinking levels.

As a result, it is thought that this research will make important contributions in seeing the studies carried out in the field from a holistic perspective. For academicians working in Van Hiele geometric thinking theory and publishing in this field, knowing the research subjects used in this field, the sample group studied, the methods, data collection tools and analysis methods will shed light on the studies to be carried out. In this context, it is thought that this research can be used as a determining resource in guiding new studies to be made.However, it should be noted that the results of the study are limited to 33 articles and 50 theses on Van Hiele geometric thinking levels between 2003 and 2020. In this direction, it can be thought that the examination of theses and articles published in different dates in the world in future studies will draw a broader picture in reflecting the development and change of Van Hiele geometric thinking levels and research conducted in Turkey.

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Appendix: Theses and Articles Examined Within The Scope Of The Research Conducted

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